Module II: Computations in the Physical World, Lecture II.b

Chi-Ning Chou @ 2022 January Mini-Course "What is Computation? From Turing Machines to Black Holes and Neurons"

Two Extremes: Quantum and Gravitational Theories

Module II: Computations in the Physical World

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."

– Albert Einstein

Last Lecture

- Newton’s laws
- Classical mechanics and computation
- Statistical mechanics and computation

- Quantum mechanics
- Quantum computing
- A glimpse on gravitational theory
- Computational views of black holes

This Lecture

The Two Extremes

Quantum Theory

Gravitational Theory

Computation

Quantum Mechanics

"Nobody understands quantum mechanics."

– Richard Feynman

Why Quantum?

Phenomena that cannot be explained by classical mechanics

In classical mechanics, the world is “continuous”, but in real world…

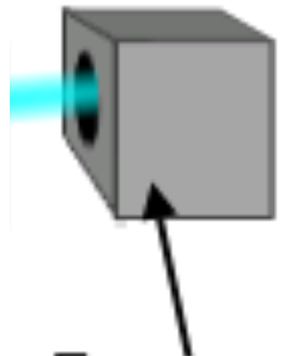
lots of physical quantities take “discrete values”

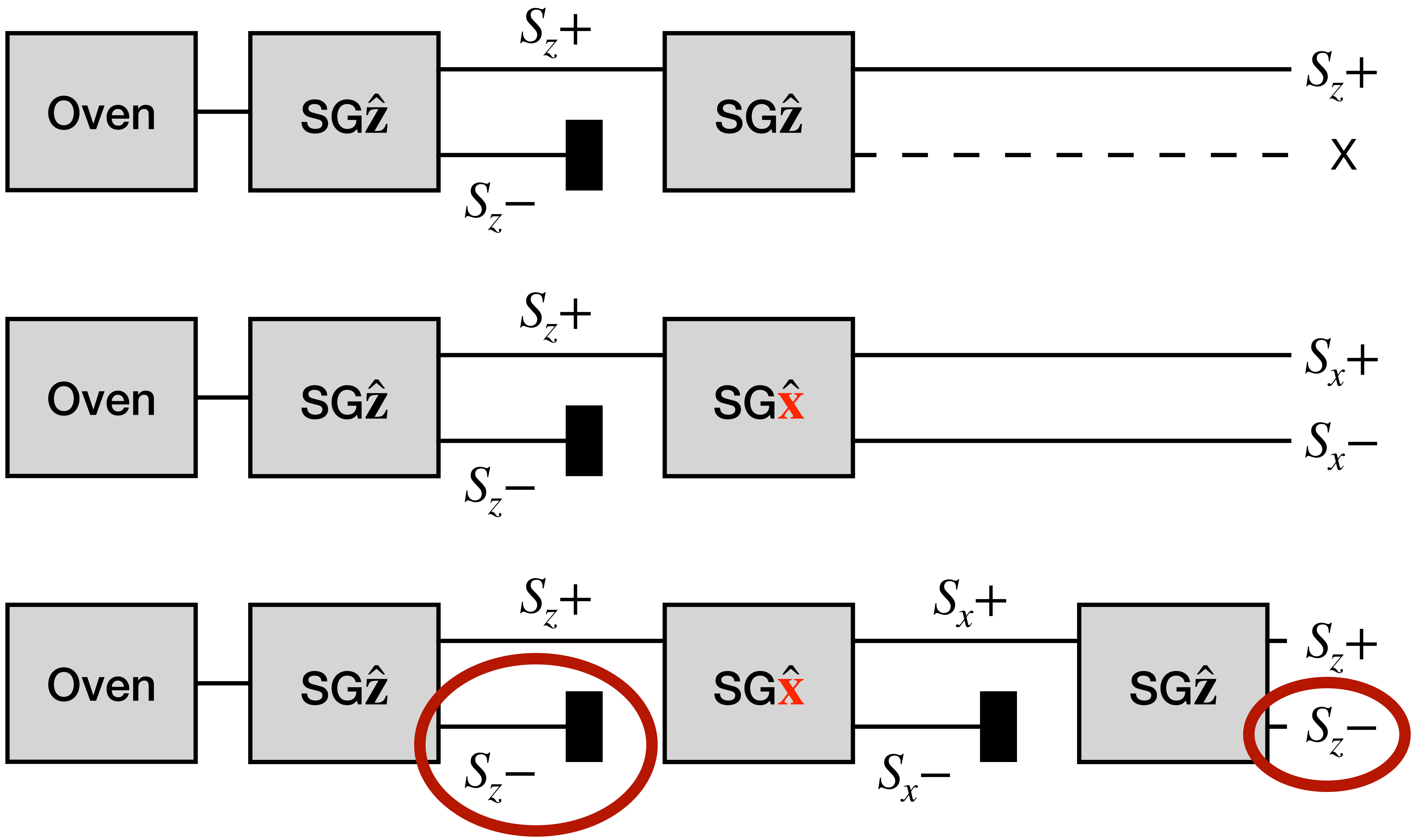
and

some objects behave like both “particle and wave”!

Quantum physics aims explain the classically unexplainable observations!

Example: the Stern-Gerlach Experiment





Quantization of angular momentum

Wave-particle duality of silver atoms

Quantum Mechanics is a Theory for Explaining these Phenomena

Quantization of Physical Quantities

Wave-Particle Duality

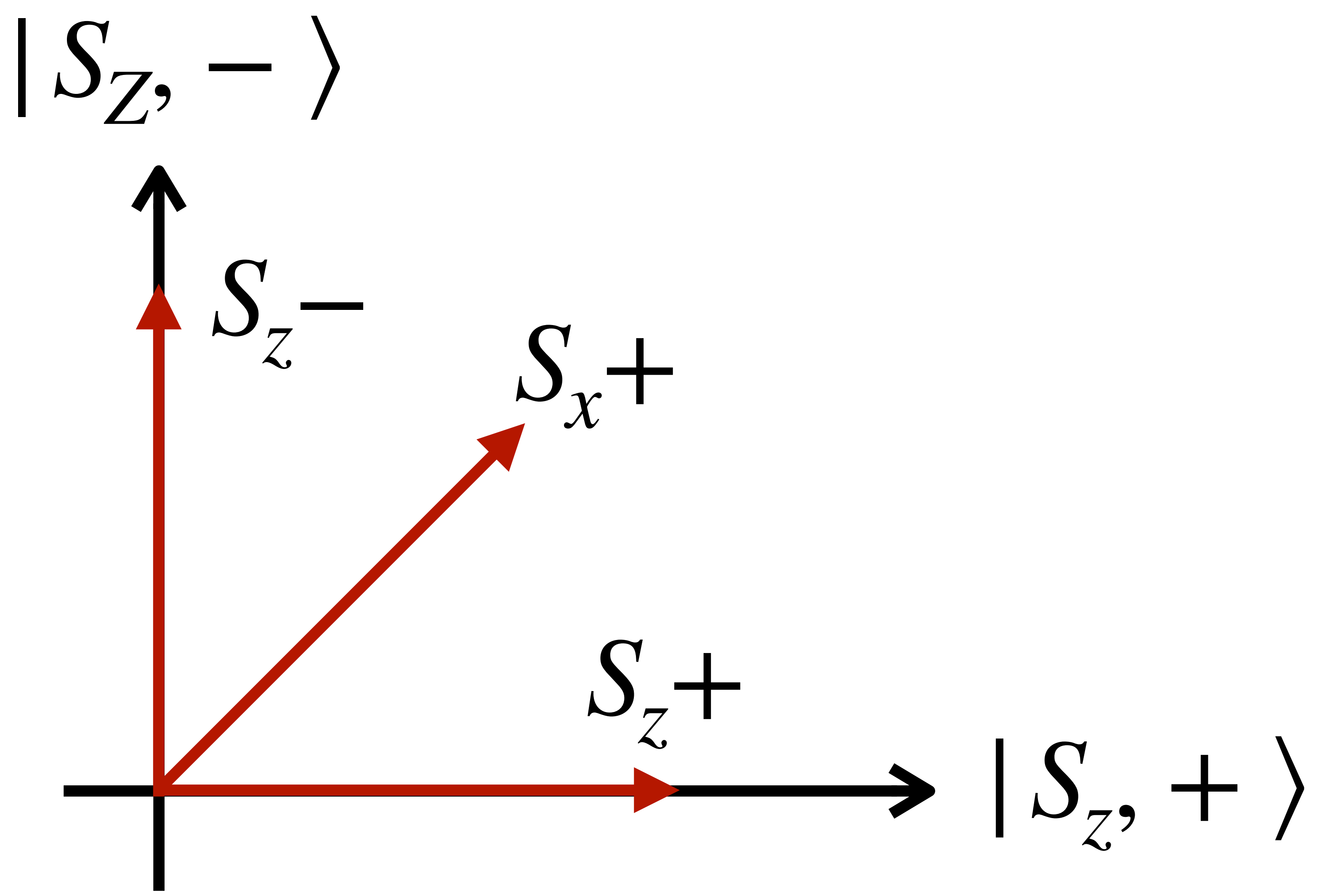
Postulates of Quantum Mechanics

From phase space to Hilbert space

Q: How to model the quantization/discreteness?

A: Use the "**coordinate**" of a mathematical space!

Example: Use the first two dimension of the space to indicate S_z+ and S_Z-.



Q: How to model the wave-particle duality?

A: Use the periodic structure of "**complex numbers**"!

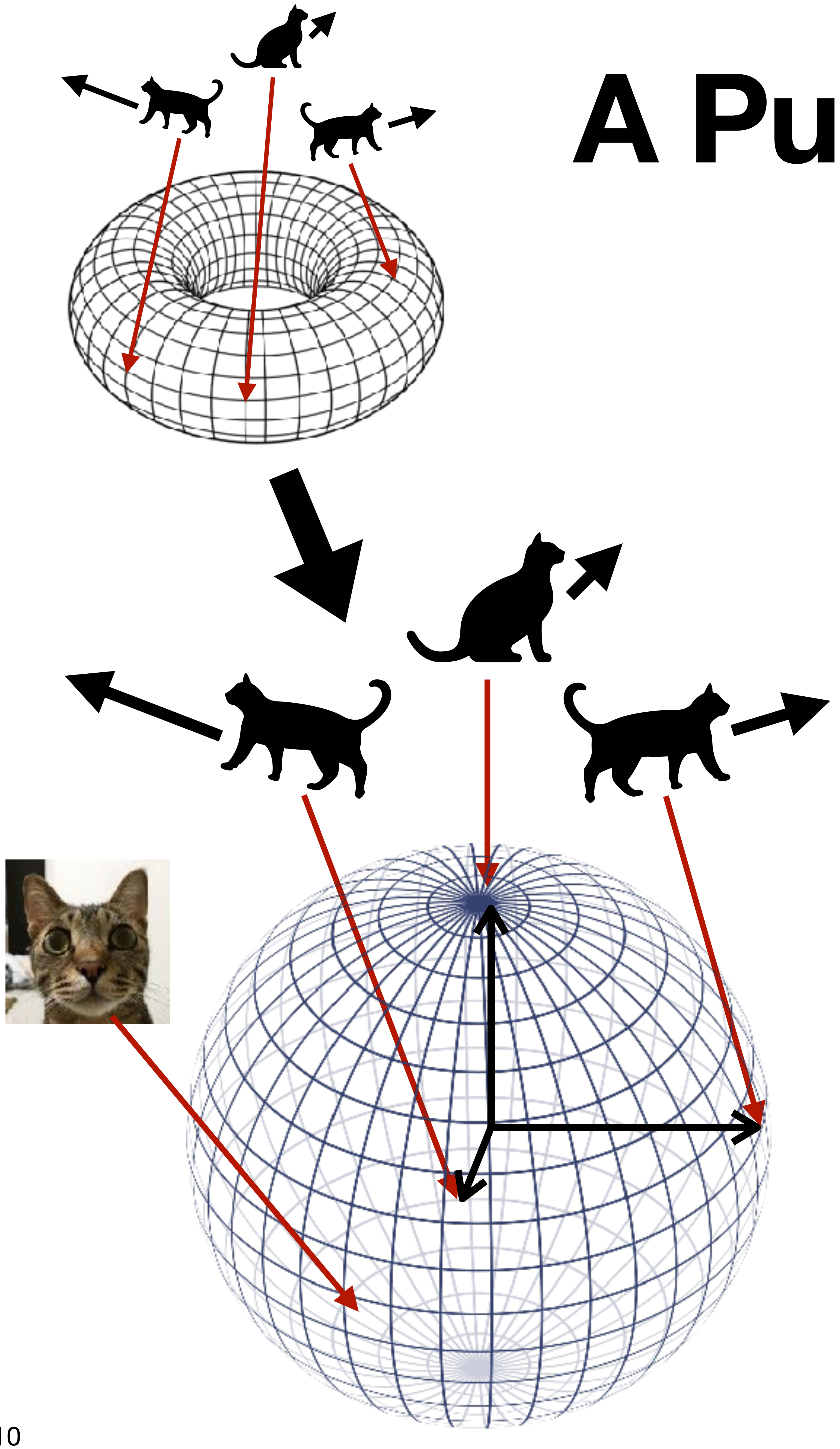
Example: Denote S_x+ as $\frac{1}{\sqrt{2}}|S_z,+\rangle + \frac{1}{\sqrt{2}}|S_z,-\rangle$ and denote S_y+ as $\frac{1}{\sqrt{2}}|S_z,+\rangle + \frac{i}{\sqrt{2}}|S_z,-\rangle$.

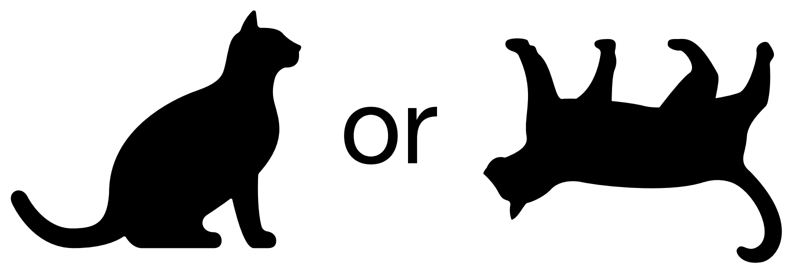
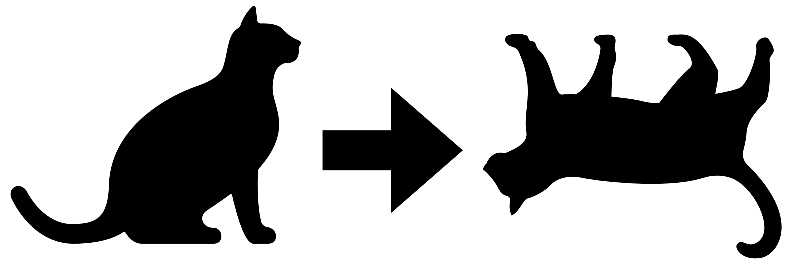
Hilbert Space

Complex vector space with an inner product structure.

* More mathematical details in advanced sections.

A Pure Mathematical World



Physics	Math	Intuition	Example
A physical measurement	An orthogonal basis	Each “dimension” encodes one possible result	The posture of a cat
(Pure) State	A point on the sphere	A point on the surface of a (high-dimensional) ball	
Evolution of a state	A unitary transformation	A rotation of the ball	

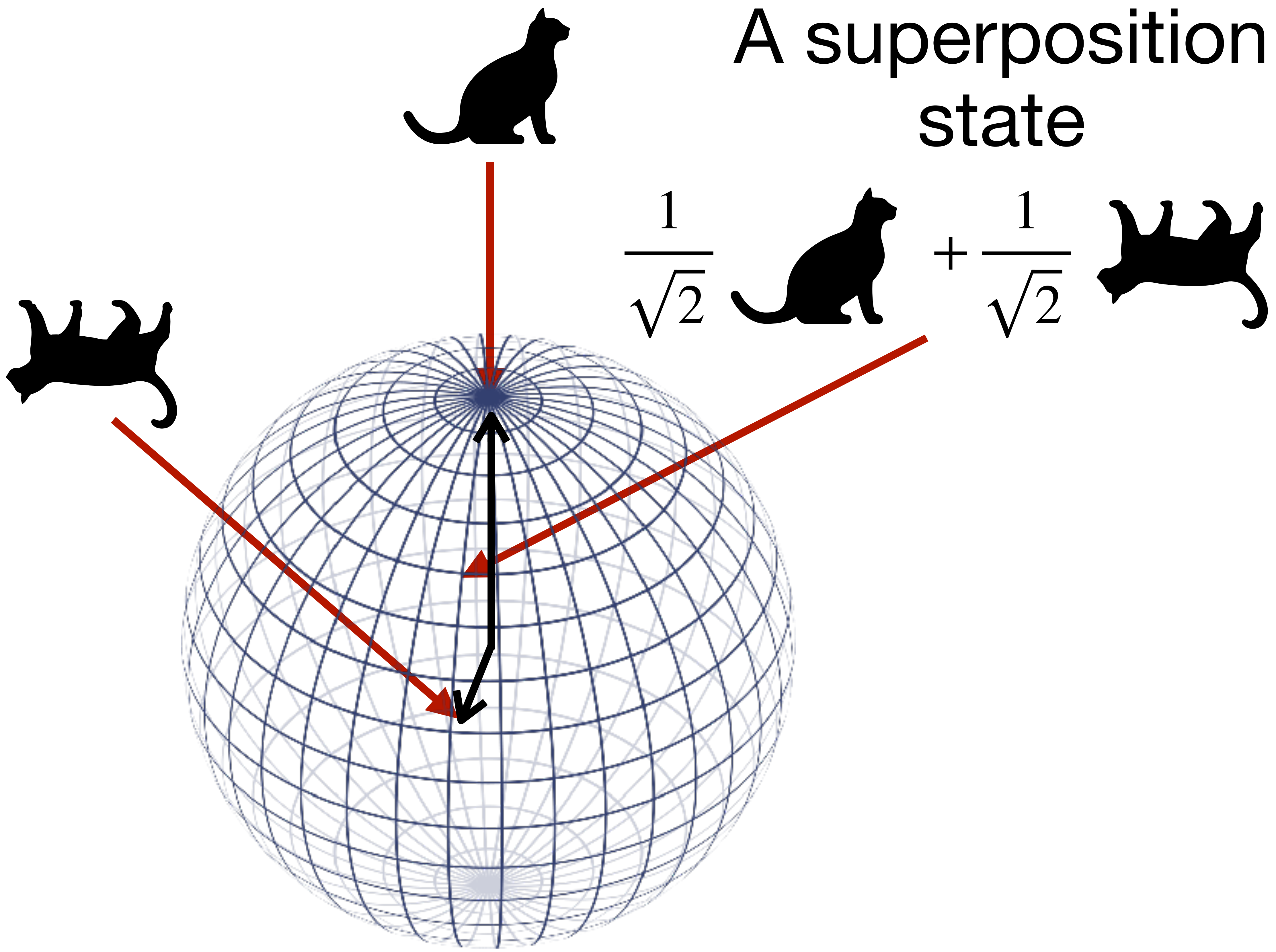
Q: What if a state is not a “basis vector”?

Basis vector

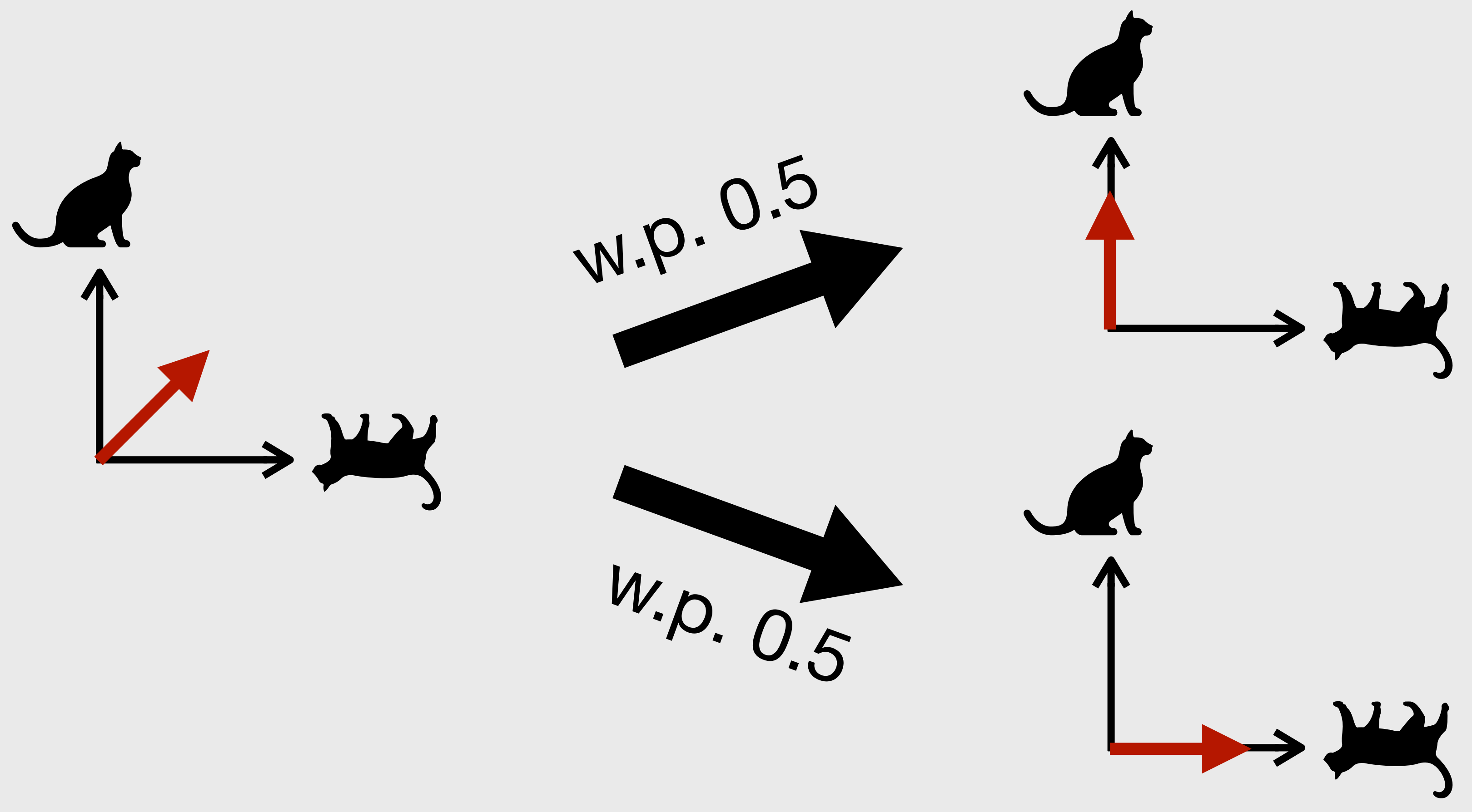
* For those who knows “Bloch sphere”, the ball here is NOT a Bloch sphere, but a simplified picture for the Hilbert space.

Quantum Superposition

An analogy of “waves” from classical physics



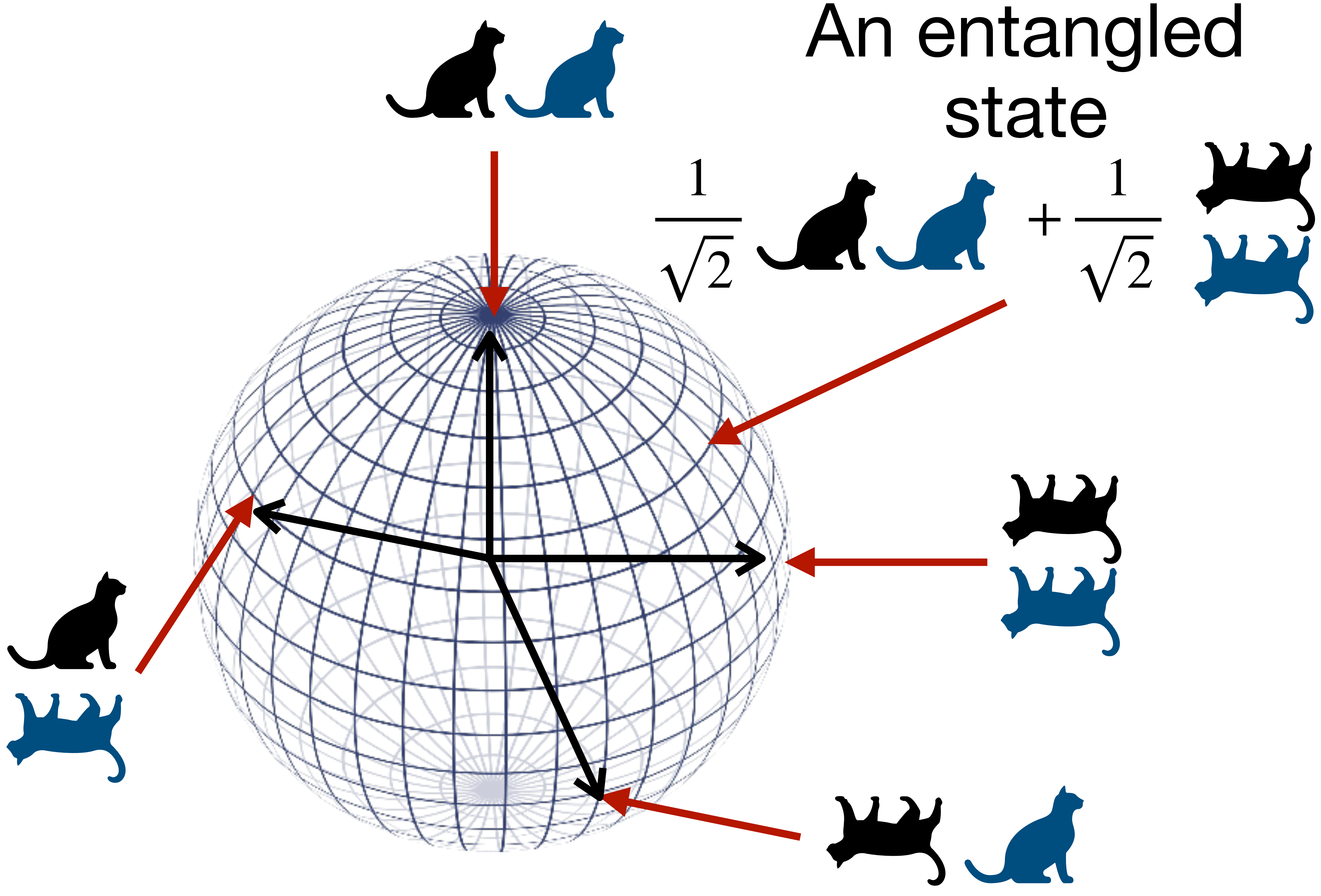
Measuring a superposition state



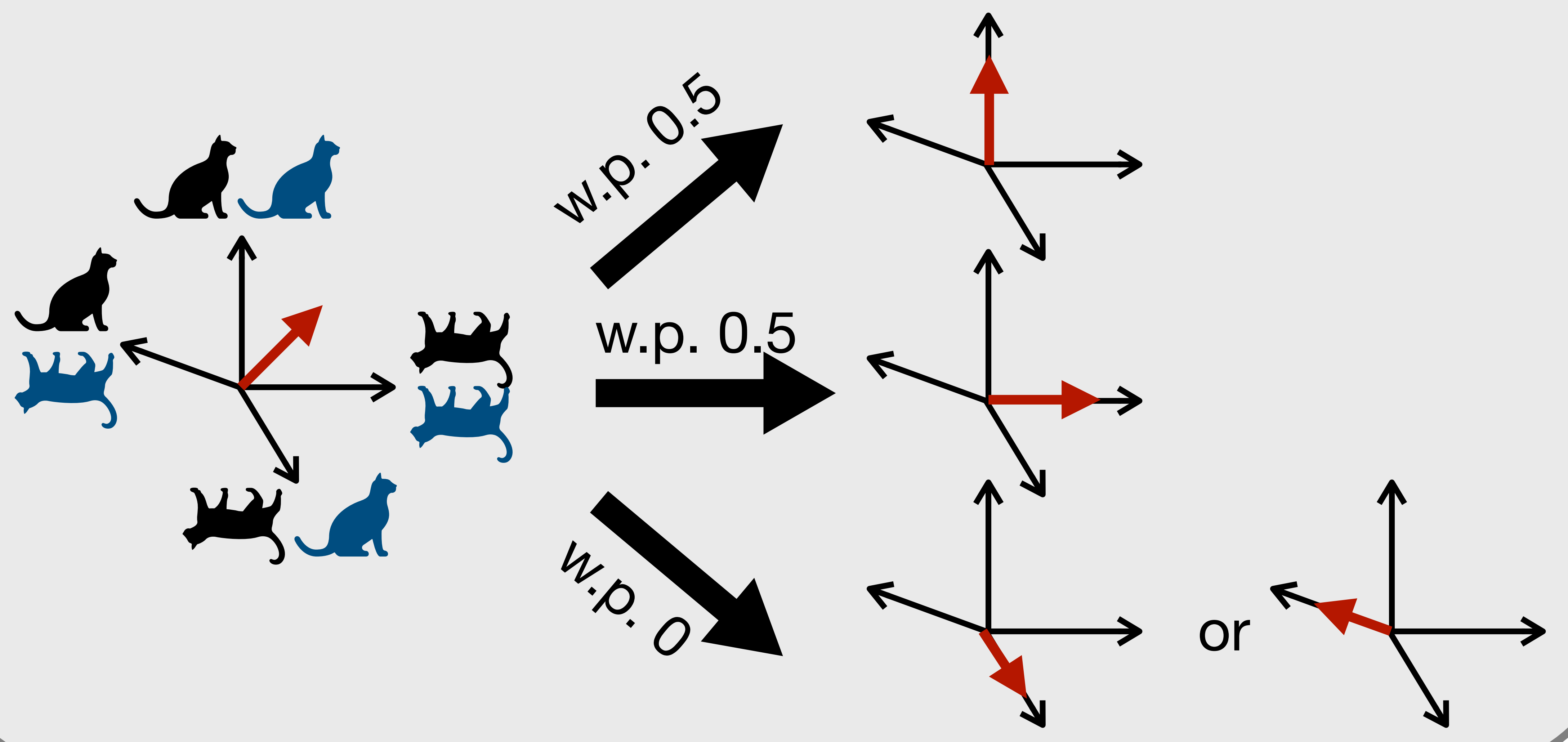
Superposition is **NOT** classical probability!!

Quantum Entanglement

A special way of “correlation” in quantum physics



Measuring an entangled state



Entanglement is **NOT** classical correlation!!

* Entangled states are s special cases of superposition states.

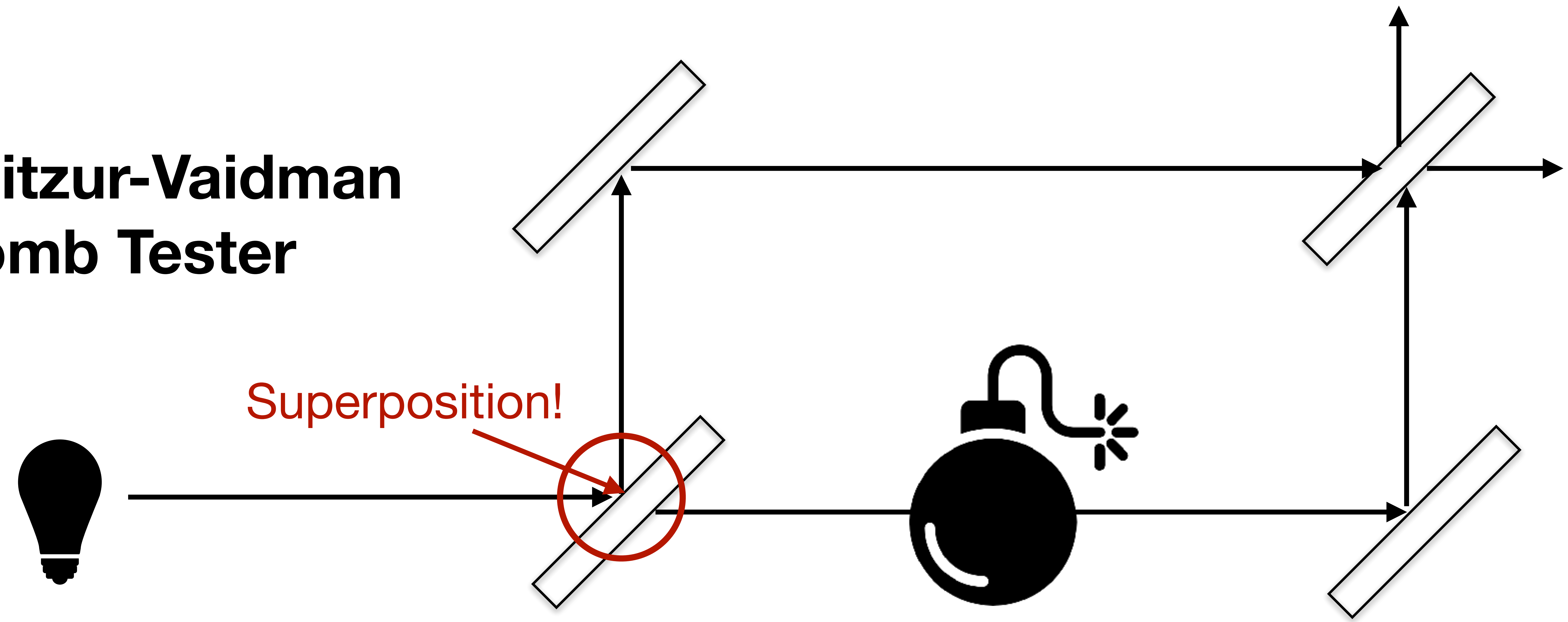
Example: The Quantum Bomb Tester

Q: How to determine if a bomb is live of dud?

Q: Can you come up with an *interaction-free* tester?

This is possible using quantum superposition!

The Elitzur-Vaidman Bomb Tester

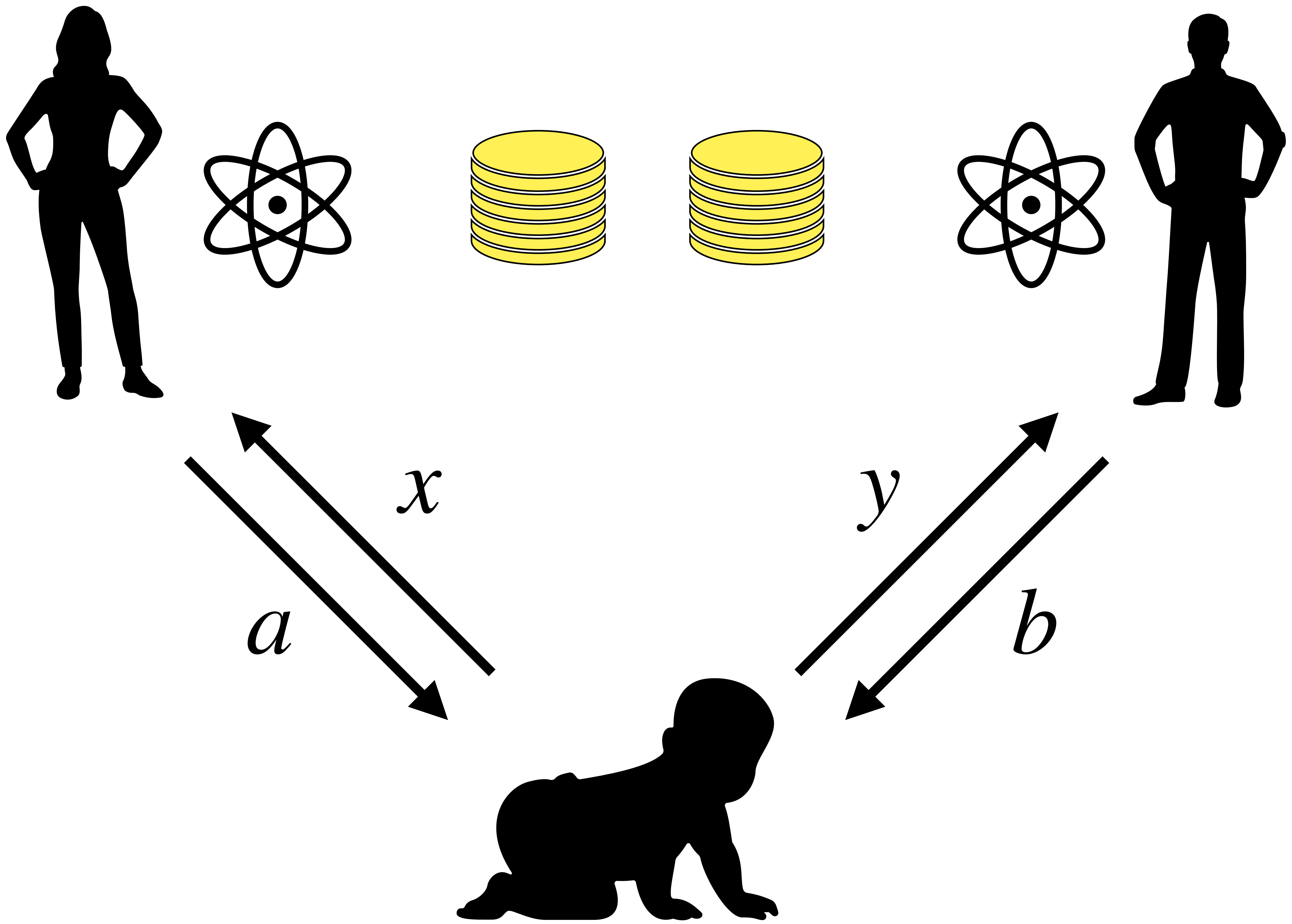


* See here for an animation to play around with!

Example: CHSH Game

x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0



(1) Flip two random coins x and y

(2) Receive a and b

(3) Check if $x \wedge y = a \oplus b$

Classical correlation

Winning probability $\leq 75\,\%$

Quantum correlation

Winning probability $\approx 85\,\%$

Xun Gao

(Jan. 13 11am-12pm ET)

Evolution of a Quantum State

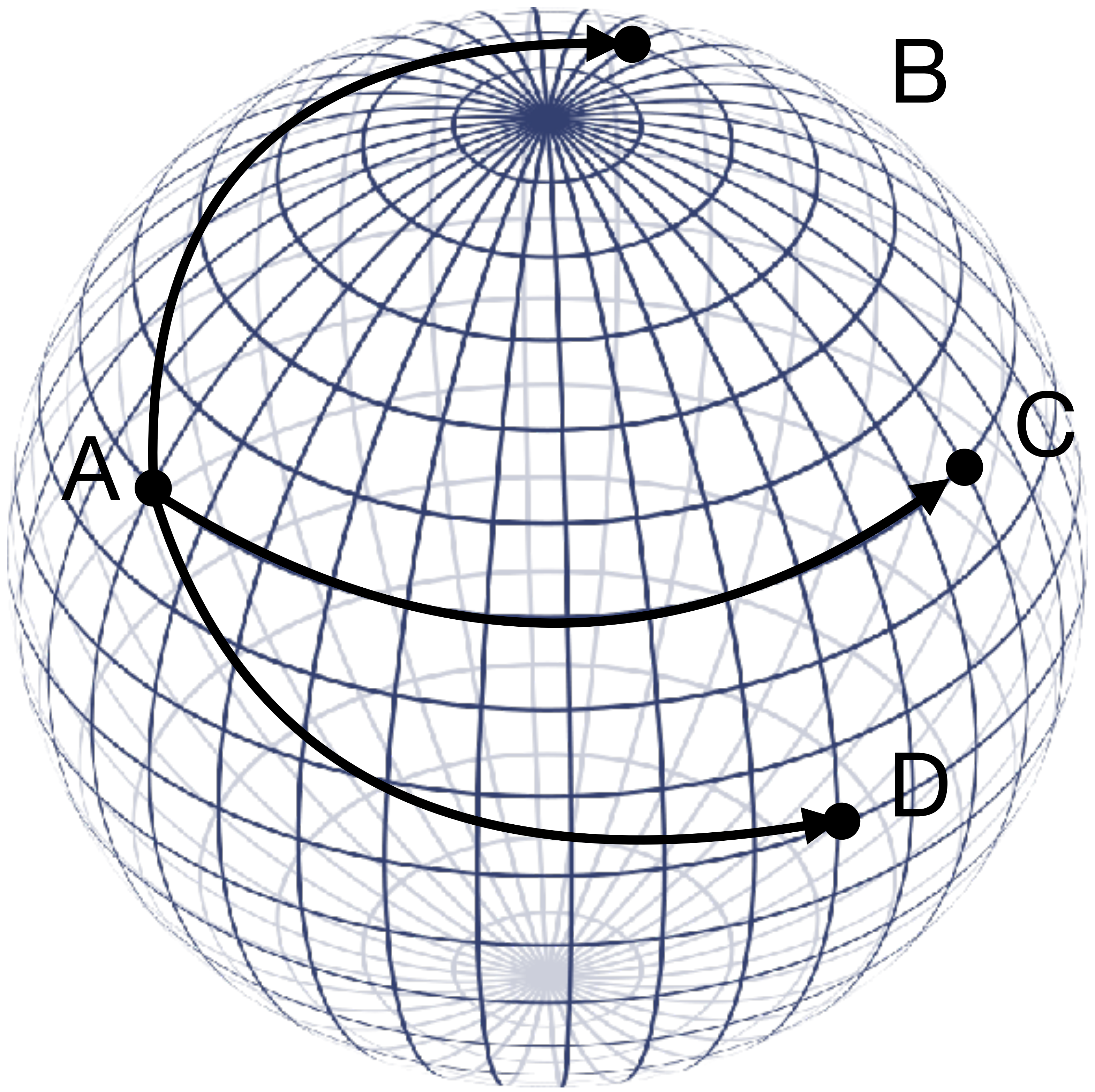
In convention, people use the bra-ket notation, e.g., $|\text{🐈}\rangle$, to denote a "quantum state".

Similar to classical mechanics, we care about the time evolution of $|\text{🐈}\rangle$.

Classical Mechanics	Quantum Mechanics
Principle of stationary action	Quantum variational principle
Lagrangian/Hamiltonian	Quantum Hamiltonian
Euler-Lagrange equation $$\frac{\partial L}{\partial \mathbf{q}} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}$$	$\boxed{\text{Schrödinger equation} \quad ih\frac{d}{dt}|\text{🐈}\rangle = H|\text{🐈}\rangle}$

* More mathematical details in advanced sections.

Path Integral View of Quantum Evolution



Q: Which path to take?

A: All of them!! Each path has a *different weight* (determined by the Hamiltonian) and the final state is a superposition of the weighted sum of them.

Q: So what?

A1: Unlike in the classical world, a state in the quantum world can evolve "in parallel"!

A2: "Interference" can take place.

Quantum Computing

"Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem because it doesn't look so easy."

– Richard Feynman

Feynman's Vision on Quantum Computing

"Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem because it doesn't look so easy."

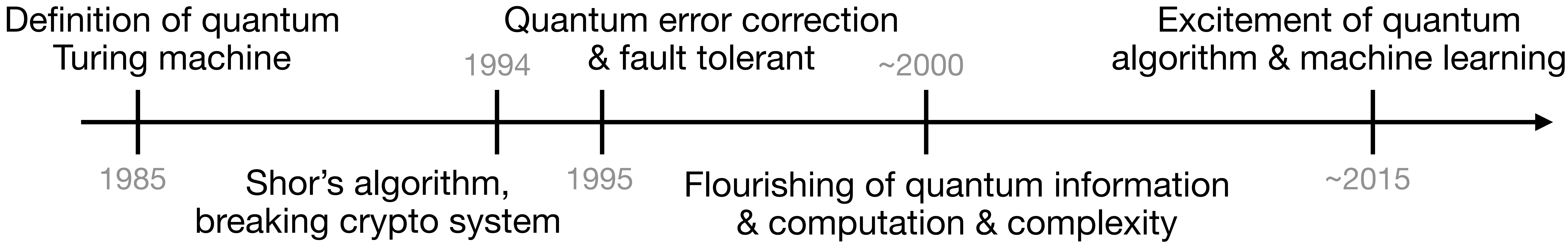
Q: Using quantum mechanics to build a fundamentally new type of computing machine for simulating quantum systems?

Q: Challenge the extended Church-Turing Thesis?

Richard Feynman 1918-1988

Beyond Simulating Quantum Systems

Quantumness as a computational resource!



Q: What type of tasks can quantumness be useful (theoretically)?

Q: In what regime can (fault tolerant) quantum computer be implemented?

Q: What's the computational mechanism and how it differs from classical?

"Quantum Correlation: the Resource to Make Quantum Machine More Powerful"

Xun Gao
(Jan. 13 11am-12pm ET)

"Quantum Machine Learning from Algorithms to Reality"

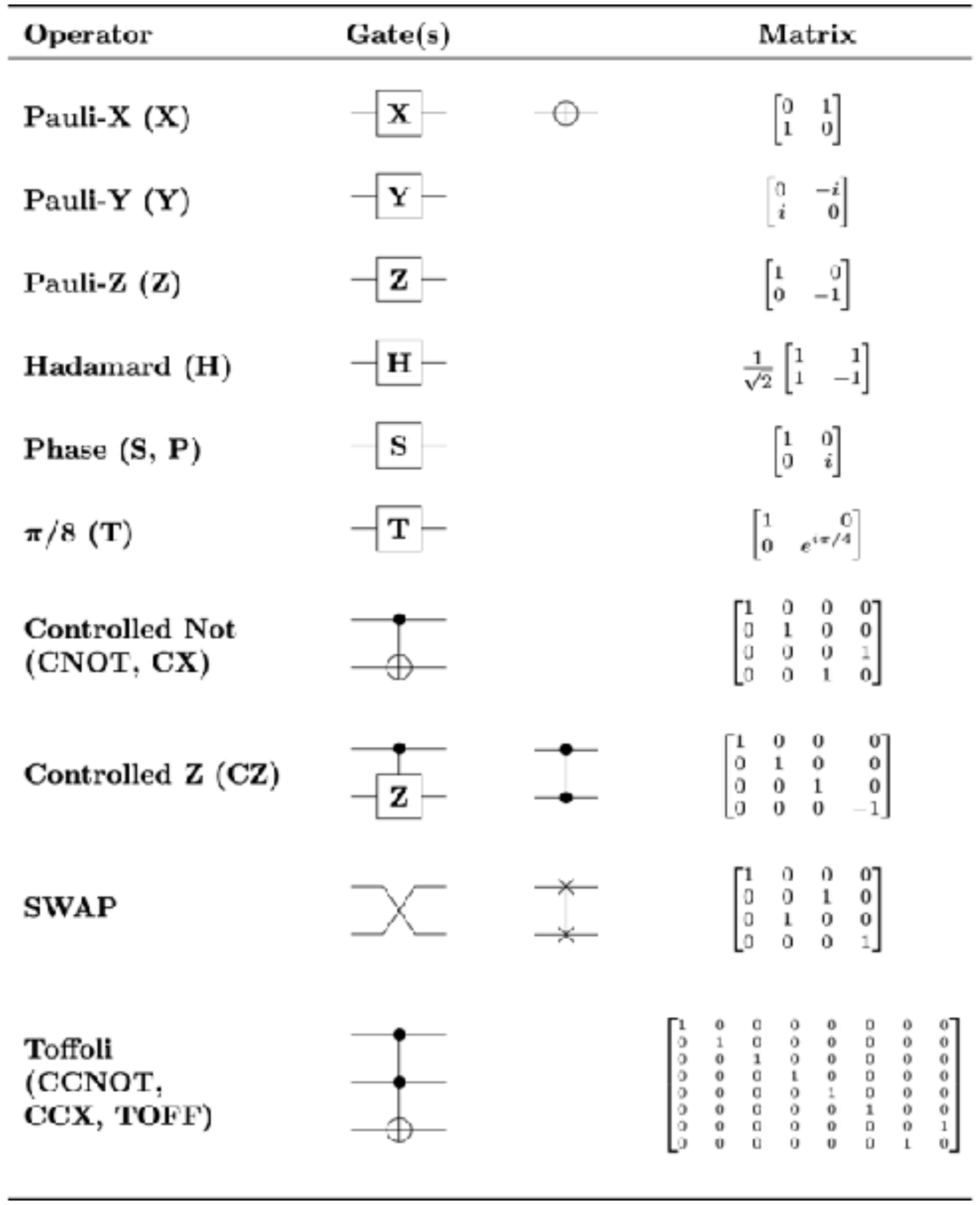
Khadijeh Sona Najafi
(Jan. 14 11am-12pm ET)

It will be a semester-long course to cover all these example….

Instead, here I decided to focus on introducing the two main "paradigms" people are using in quantum computing!

Example 1: Quantum Circuits

A “quantumization” of boolean circuits



Operator	Gate(s)	Matrix
Pauli-X (X)		$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Phase (S, P)		$\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$
$\pi/8$ (T)		$\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$
Controlled Z (CZ)		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$
SWAP		$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$

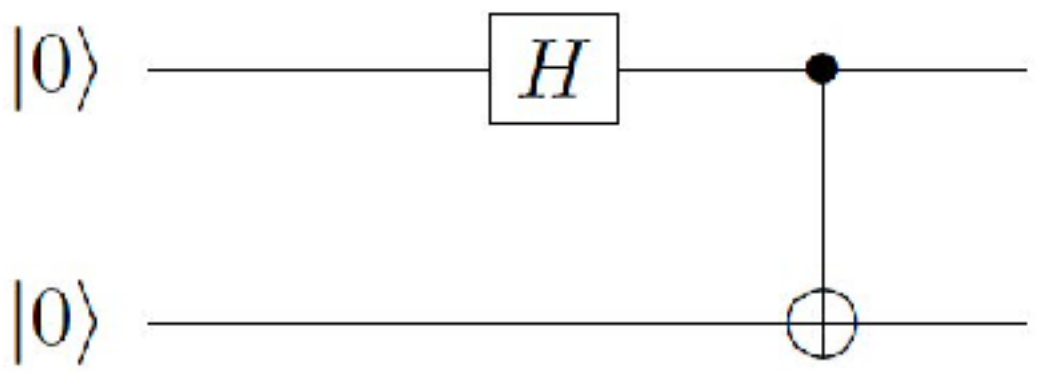
Qubits: "quantum bits”.

- A state of n qubits has dimension 2^n (each n -bit string uses one degree of freedom).

Quantum gates: beyond Boolean operations!

- Create superpositions and entanglements.

Quantum circuits: sequentially applying quantum gates on qubits.



The output of a quantum circuit is a quantum state instead of 0/1!

Example 2: Adiabatic Quantum Computation

Configure a computation into a quantum system!

Instead of applying quantum gates, use the time evolution on state $|\Psi(t)\rangle$.

Hamiltonian: Specify the "evolution" of a quantum state.

$$i h \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

(Schrödinger equation)

Hamiltonian

Local Hamiltonian: Specify the "evolution" of a subset of qubits.

$$H = \sum_{i=1}^{m} H_i$$

Local Hamiltonian

Adiabatic Theorem

If the evolution is slow enough, then $|\Psi(t)\rangle$ goes to a lowest energy state.

Pick the local Hamiltonians properly, so that the lowest energy state is the output you want!

Quantum Computational Advantage?

A challenge to the extended Church-Turing Thesis

Extended Church-Turing Thesis

All feasible computation in the physical world can be done by a Turing machine efficiently.

2017 IBM **50** qubits

2018 Intel **49** qubits

2019 IBM **53** qubits

2019 Google **53** qubits

2020 IBM **65** qubits

2020 USTC **60** qubits

2021 USTC **66** qubits

2021 IBM **127** qubits

2021 Rydberg **256** qubits

2022 ???

More on recent exciting developments in Guest Talk II.a and II.b!

Excitement in Quantum Computing!

Be careful with the information, and make your own judgement!

“Quantum Correlation: the Resource to Make Quantum Machine More Powerful”

Xun Gao
(Jan. 13 11am-12pm ET)

“Quantum Machine Learning from Algorithms to Reality”

Khadijeh Sona Najafi
(Jan. 14 11am-12pm ET)

Sowmya
(Jan. 18
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“Basic of Quantum Computing and Future Direction”

Avantika
(Jan. 18
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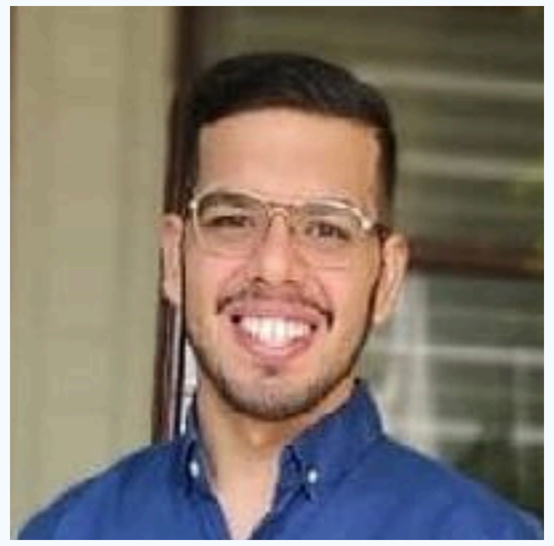
“Quantum Complexity Theory”



Kartikeya
(Jan. 20
9am-10am ET)

“Quantum Computing from a Condensed Matter Perspective”

Black Holes and Computation

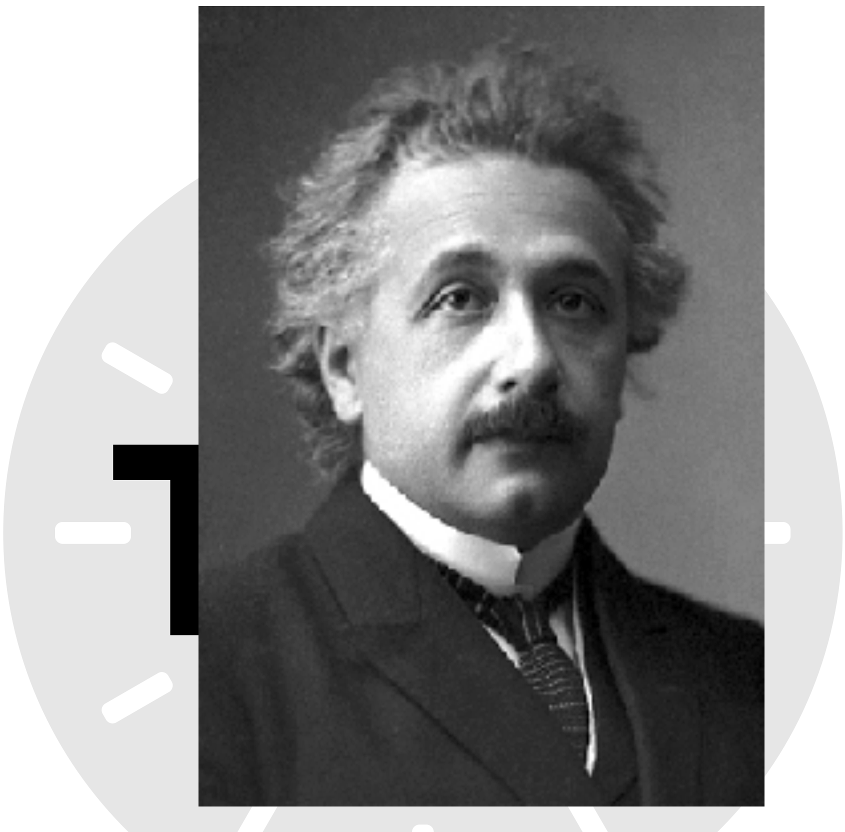
"Black holes are where God divided by zero."

– Albert Einstein

Theories for Space and Time

ce

Spacetime



T

Hermann Minkowski
1864-1909

Albert Einstein
1879-1955

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Curvature of the spacetime = **Flux of energy & momentum**

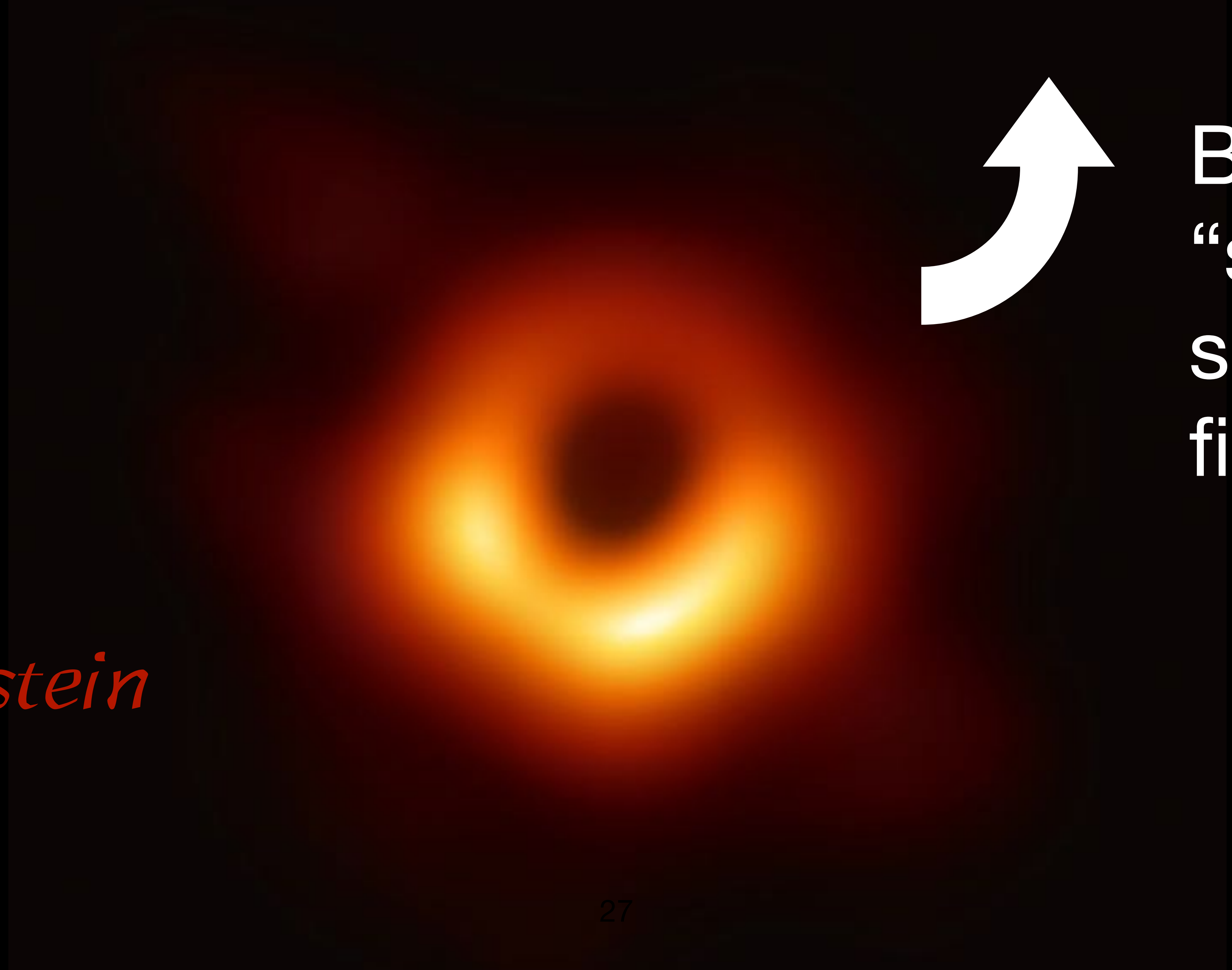
"Space-time tells matter how to move; matter tells space-time how to curve."

– John Wheeler

Black Holes

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

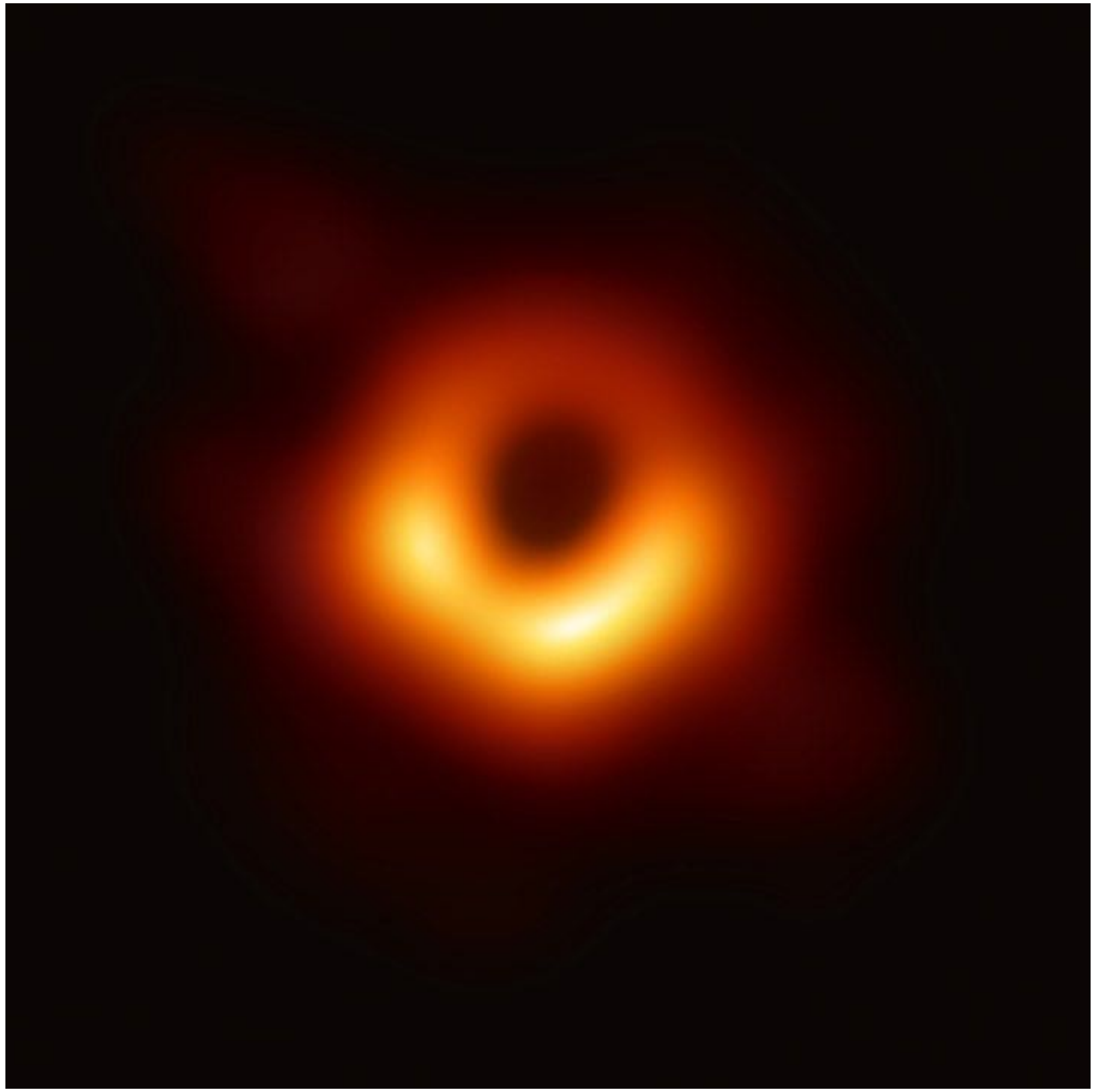
Black Holes are simply "special solutions" (a.k.a. singularities) to Einstein field equations!



"Black holes are where God divided by zero."

– Albert Einstein

Why Black Holes are Black?



Because gravity becomes too large so that even light cannot escape from a black hole!

Q: Why we should care about black holes?

A: They are “predicted” by Einstein’s relativity theory. So their existence related to the validity of Einstein’s theory.

Q: Now that scientists captured a black hole, what’s next?

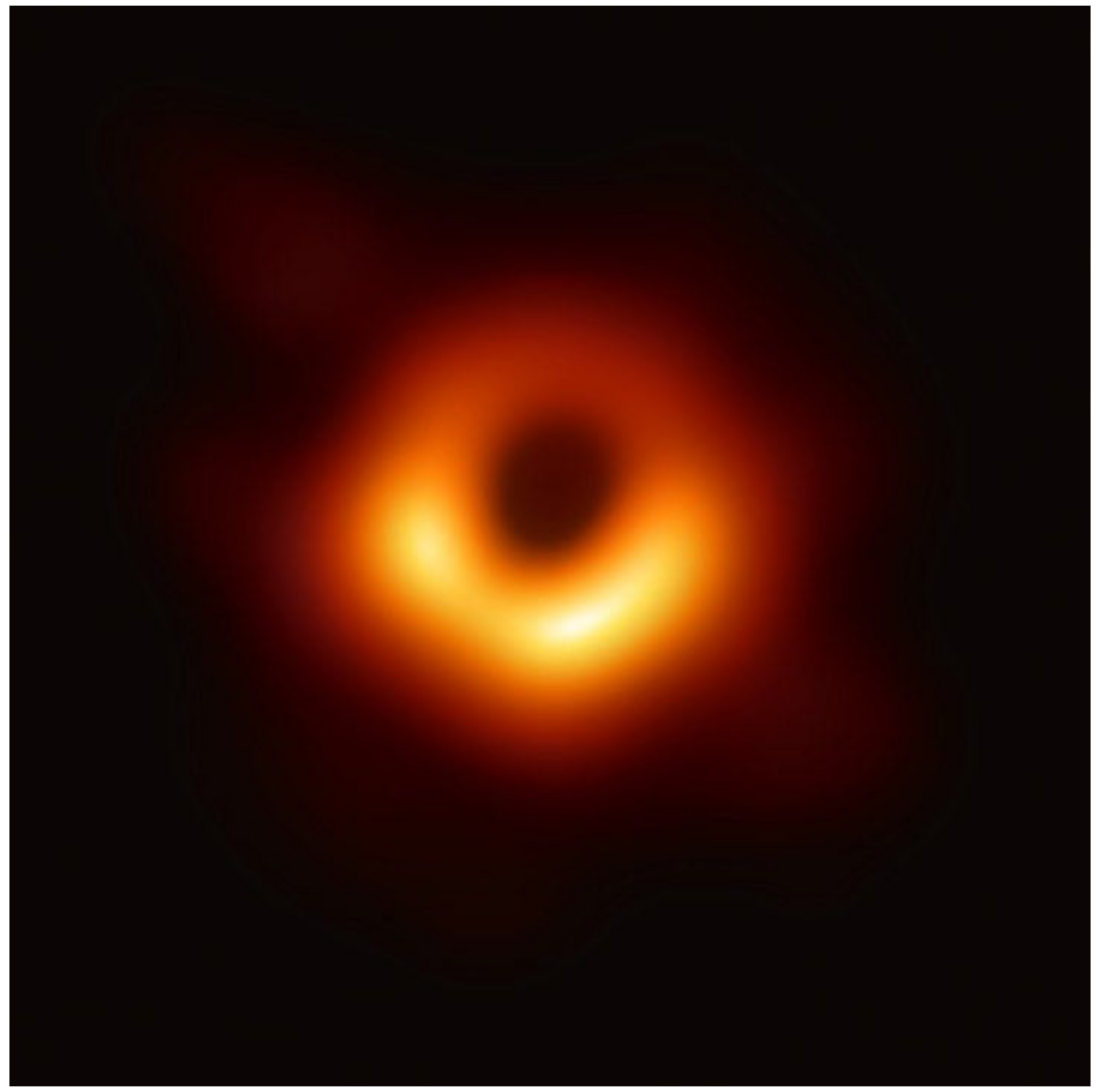
A: Unification with quantum and lots of “paradoxes” and “puzzles” to be solved…

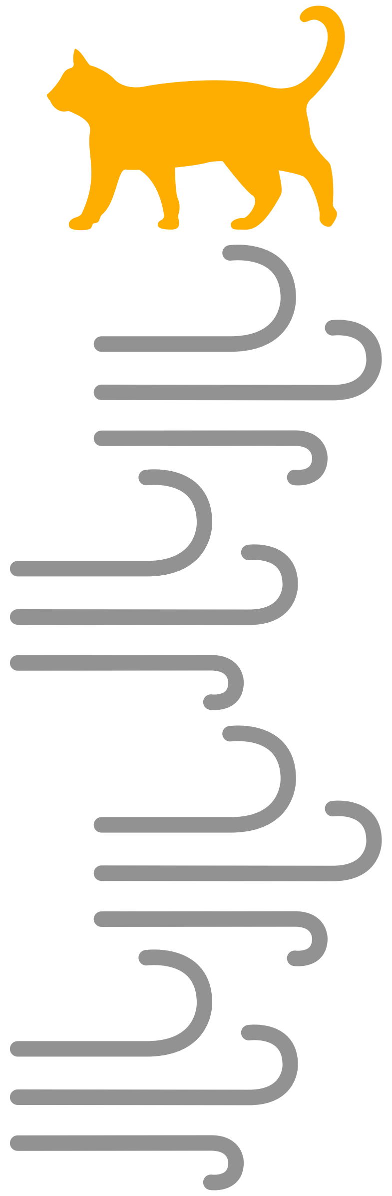
Example: Information Paradox

Black holes will evaporate!

Wait, information will also lost!?

Stephen Hawking
1942-2018





Only depends on macroscopic parameters according to the no-hair theorem!

No-Hair Theorem

A black hole can be completely characterized by its mass, electrical charge, and angular momentum.

In quantum theory, information won't lost! But no-hair theorem + Hawking radiation suggest so…

Q: Does information really lost?

Q: How to unify quantum and gravity?

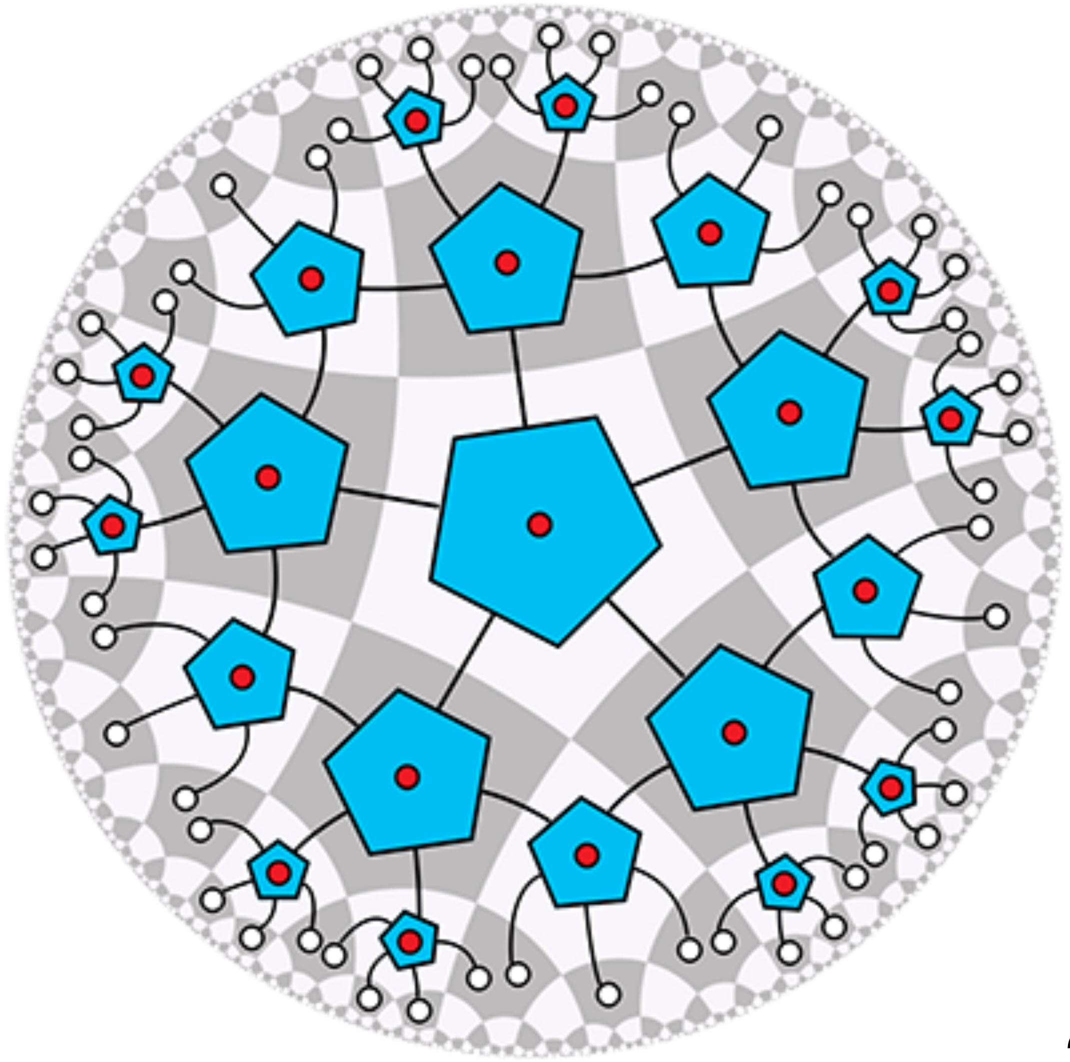
Computation as a New Angle for Gravity?

Black holes as…

Information Scramblers

Black holes "look very random"!? (pseudorandom)

Error-Correcting Codes

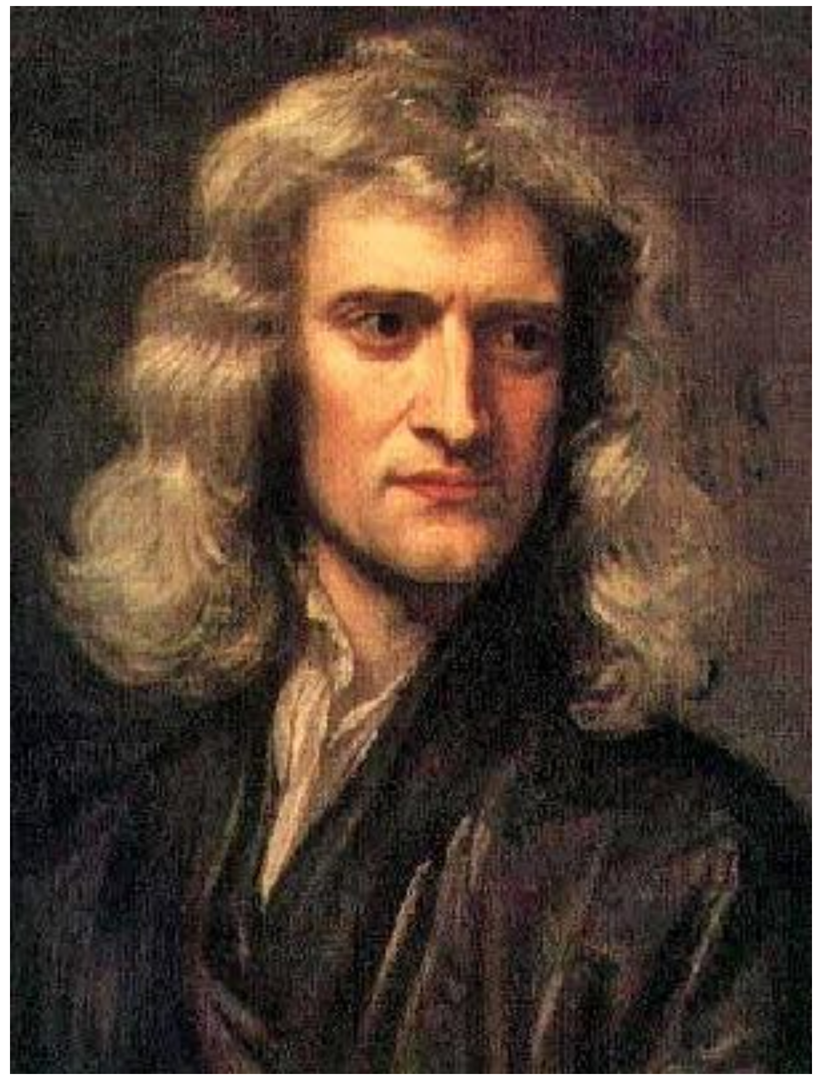


The interior of a black hole can be "locally decoded"!?

, and more…?

Properly explaining these requires some prerequisites, I might offer an advanced section on this if enough people are interested!

Put the World Views Together

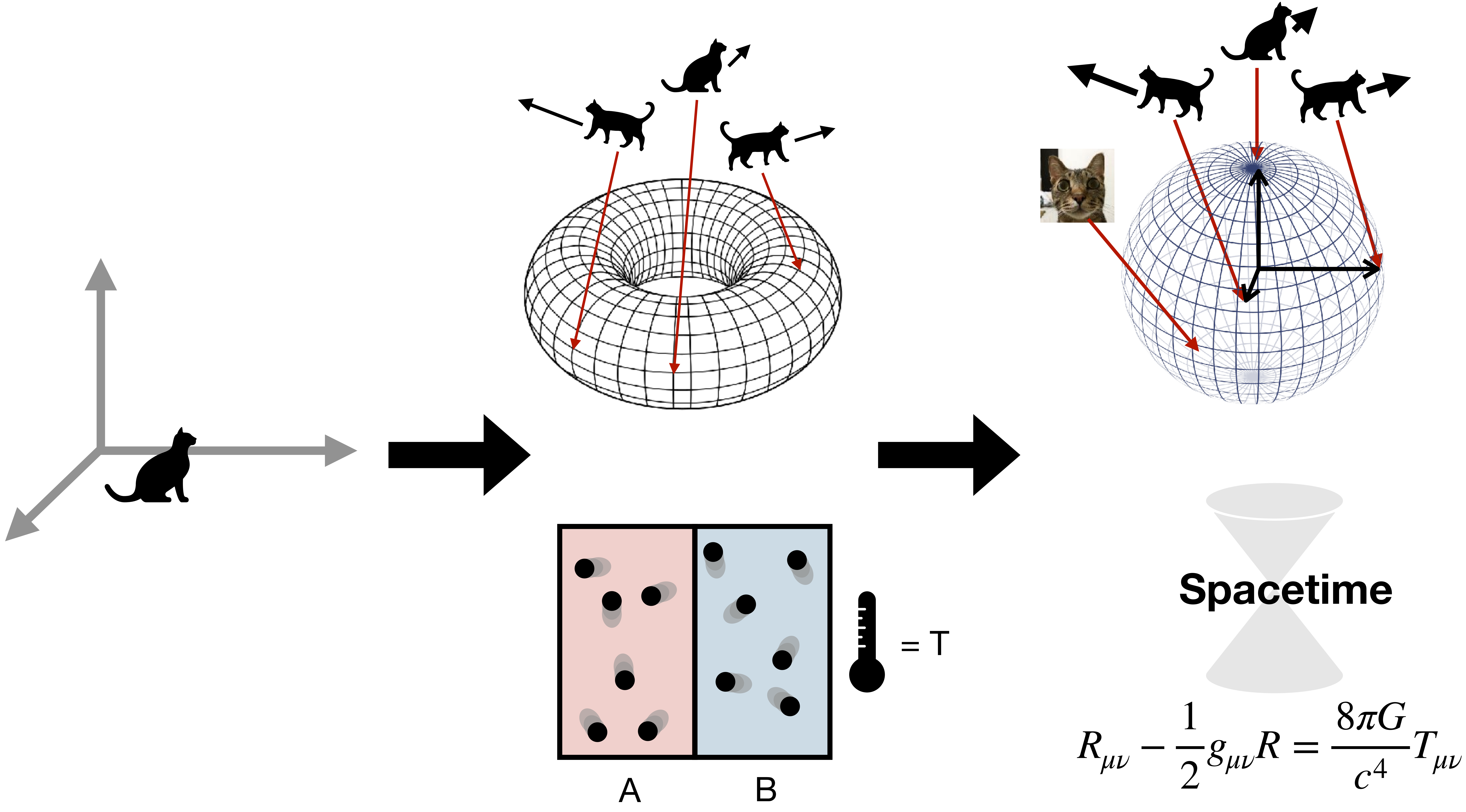


Classical Mechanics

Statistical Mechanics

Quantum Mechanics

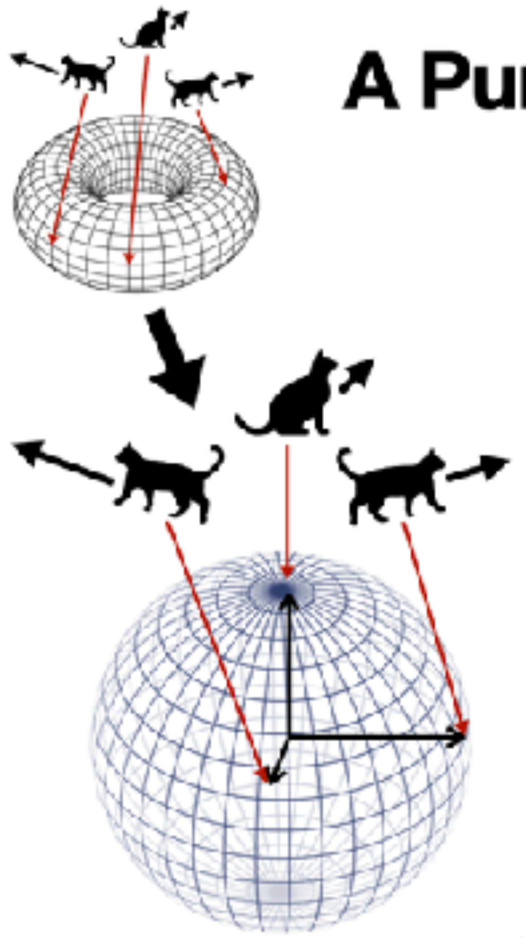
Gravitational Theory

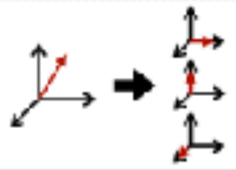

A
B
= T
Spacetime
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$

Summary

Key Concepts

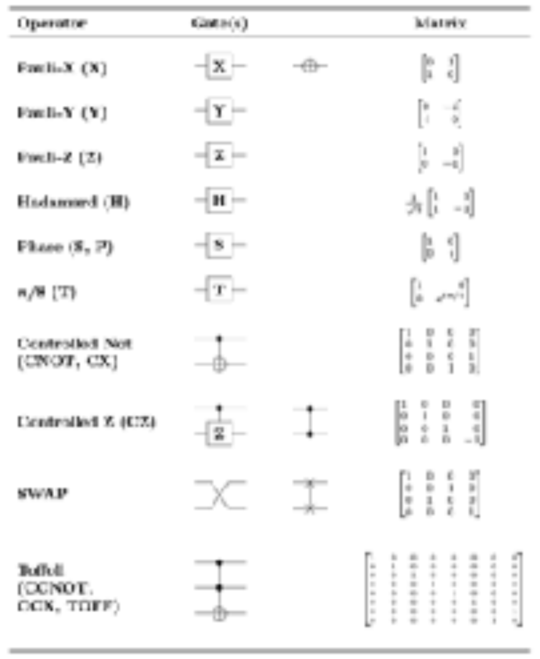
A Pure Mathematical World



Physics	Math	Intuition
A physical quantity	An orthogonal basis	Each "dimension" encodes one possible result
(Pure) State	A point on the sphere	A point on the surface of a (high-dimensional) ball
Evolution of a state	A unitary transformation	A rotation of the ball
Measuring a physical quantity	Projecting the point to an orthogonal basis	 w.p. 0.3 w.p. 0.5 w.p. 0.2

* For those who knows "Bloch sphere", the ball here is NOT a Bloch sphere, but a simplified picture for the Hilbert space.

Example 1: Quantum Circuits



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- A state of n qubits has dimension 2^n (each n-bit string uses one degree of freedom).

Quantum gates: beyond Boolean operations!

- Create superpositions and entanglements.

Quantum circuits: sequentially applying quantum gates on qubits.

The output of a quantum circuit is a quantum state, which defines a distribution over the set of n-bit strings!

Example 2: Adiabatic Quantum Computation

Instead of applying quantum gates, use the time evolution on state $|\Psi(t)\rangle$.

Hamiltonian: Specify the "evolution" of a quantum state.

$$ih\frac{d}{dt}|\Psi(t)\rangle = \underset{\text{Hamiltonian}}{H}|\Psi(t)\rangle \qquad \text{(Schrödinger equation)}$$

Local Hamiltonian: Specify the "evolution" of a subset of qubits.

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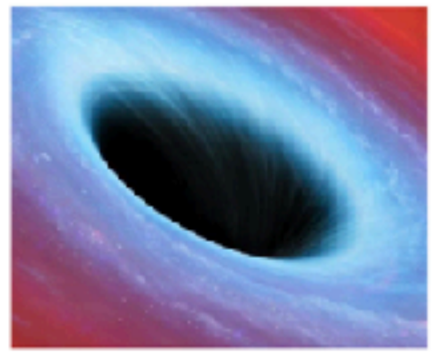
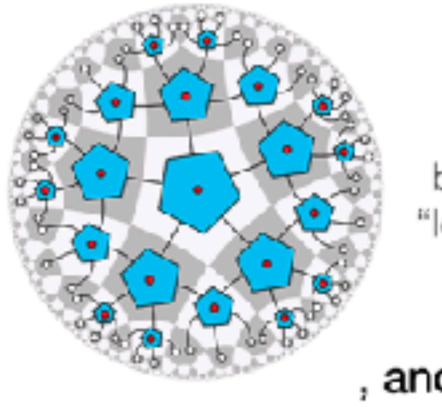
If the evolution is slow enough, then $|\Psi(t)\rangle$ goes to a lowest energy state.

Computation as a New Angle for Gravity?

Black holes as...

Information Scramblers	Error-Correcting Codes
Black holes "look very random"!? (pseudorandom) 	 The interior of a black hole can be "locally decoded"!?

, and more…?

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Guest Speakers for Module II

Xun Gao
(Jan. 13 11am-12pm ET)

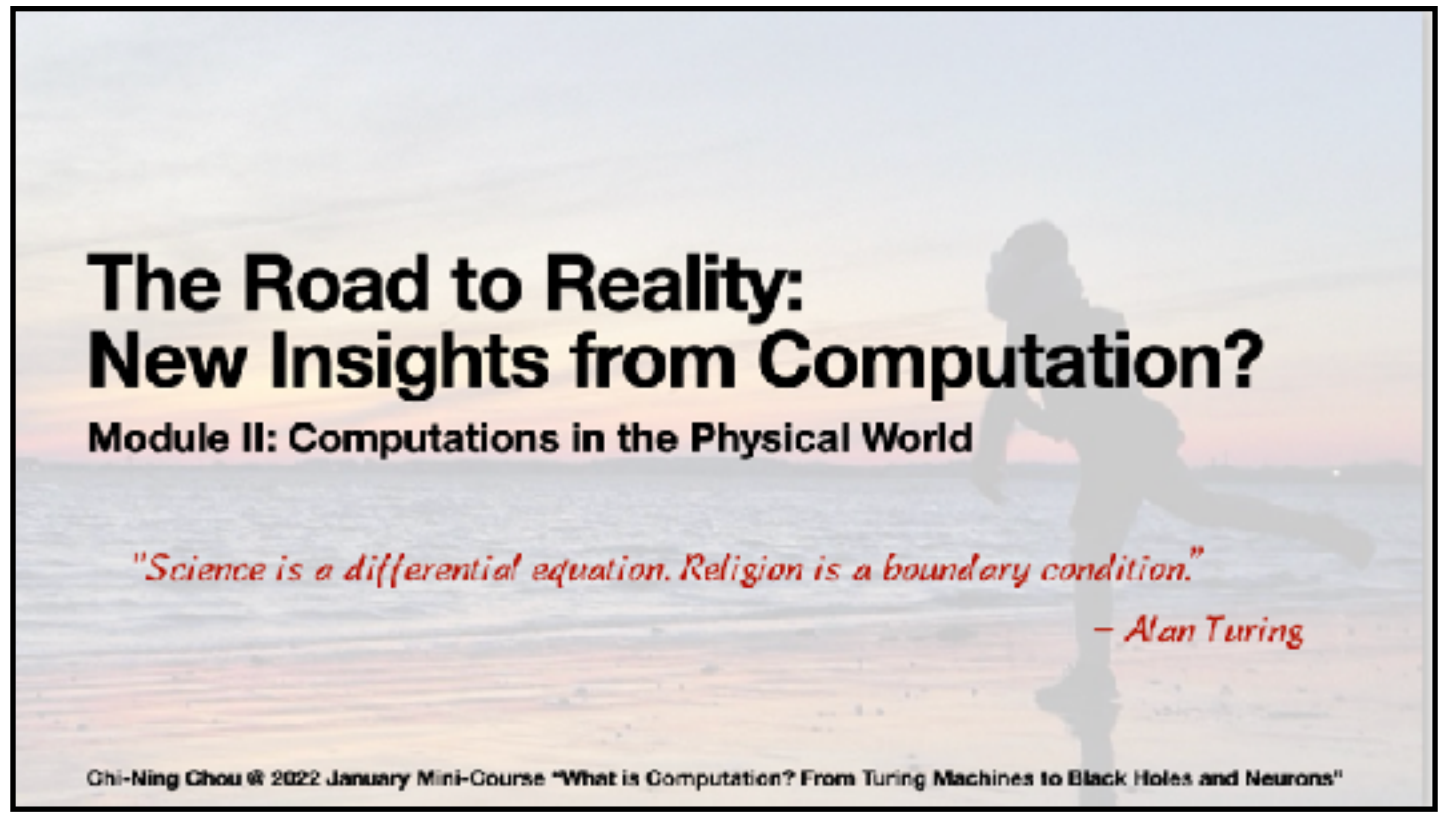
"Quantum Correlations: the Resources Making Quantum Machines More Powerful"

The recording won't be made public!

"Quantum Machine Learning from Algorithms to Reality"

Khadijeh Sona Najafi
(Jan. 14 11am-12pm ET)

Next



Lecture II.c
(Jan. 19 10am-10:50am ET)

Erick
(Jan. 13
2pm-3pm ET)

"Information Geometry"

Simone
(Jan. 14
2pm-3pm ET)

"Simulated Annealing"

Sowmya
(Jan. 18
10am-11am ET)

"Basic of Quantum Computing and Future Direction"

Avantika
(Jan. 18
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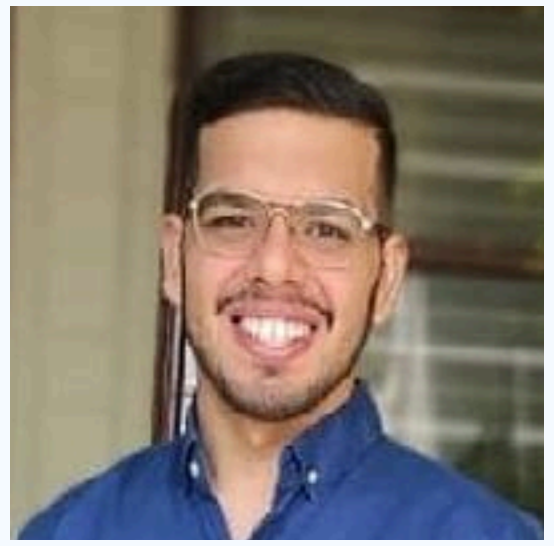
"Quantum Complexity Theory"



Kartikeya
(Jan. 20
9am-10am ET)

"Quantum Computing from a Condensed Matter Perspective"

Food for Thought

Q: What’s the most striking phenomena in the quantum world to you? (Maybe also answer this after Xun’s and Sona’s talk)

Q: In the quantum bomb tester and the CHSH game, how does quantumness play a role?

Q: Why no-hair theorem intuitively make sense?

Exercise

- Work out and details of the quantum bomb tester and the CHSH game.
- Find a news about quantum computing on media and try to see if there’s any misunderstanding in it! Do the same thing for black holes.
- Try to figure out why Einstein said “black holes are where god divided by zero.”

References

Articles:

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- Mézard, Marc, Giorgio Parisi, and Miguel Angel Virasoro. Spin glass theory and beyond: An Introduction to the Replica Method and Its Applications. Vol. 9. World Scientific Publishing Company, 1987, link.
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