"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality"

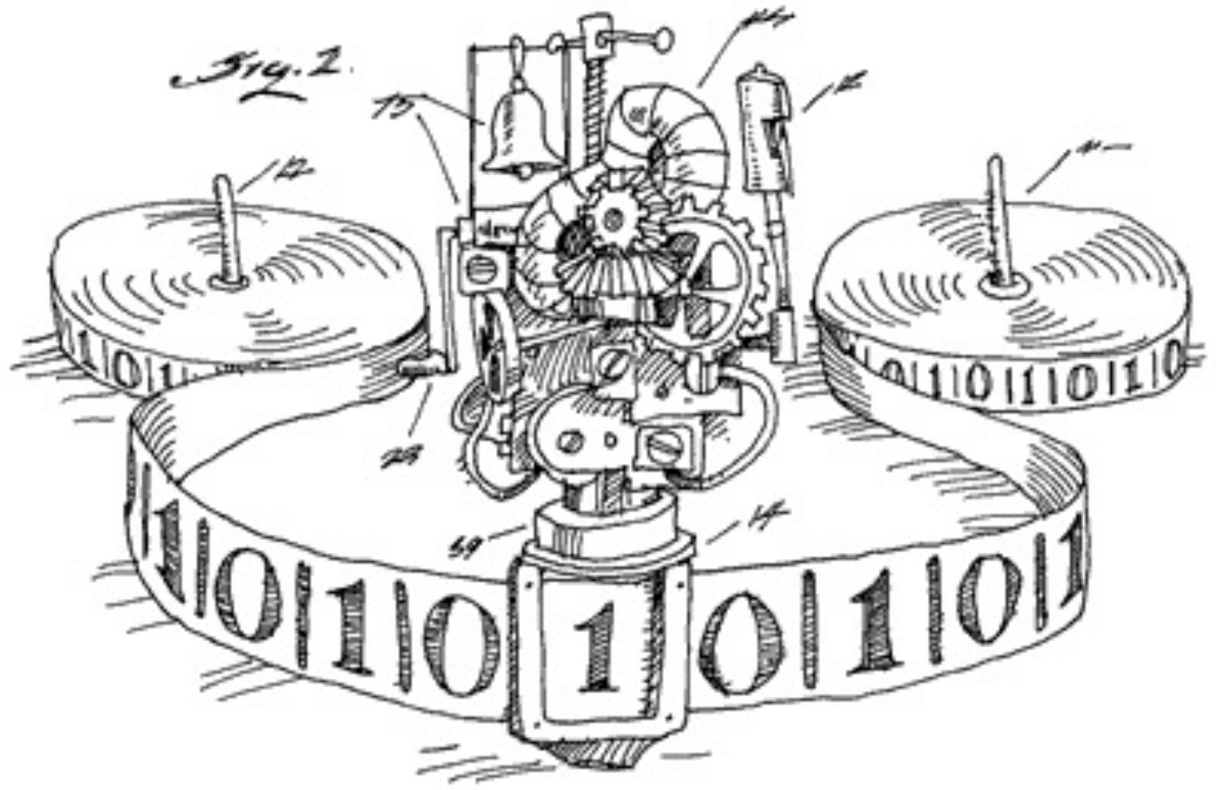
– Albert Einstein

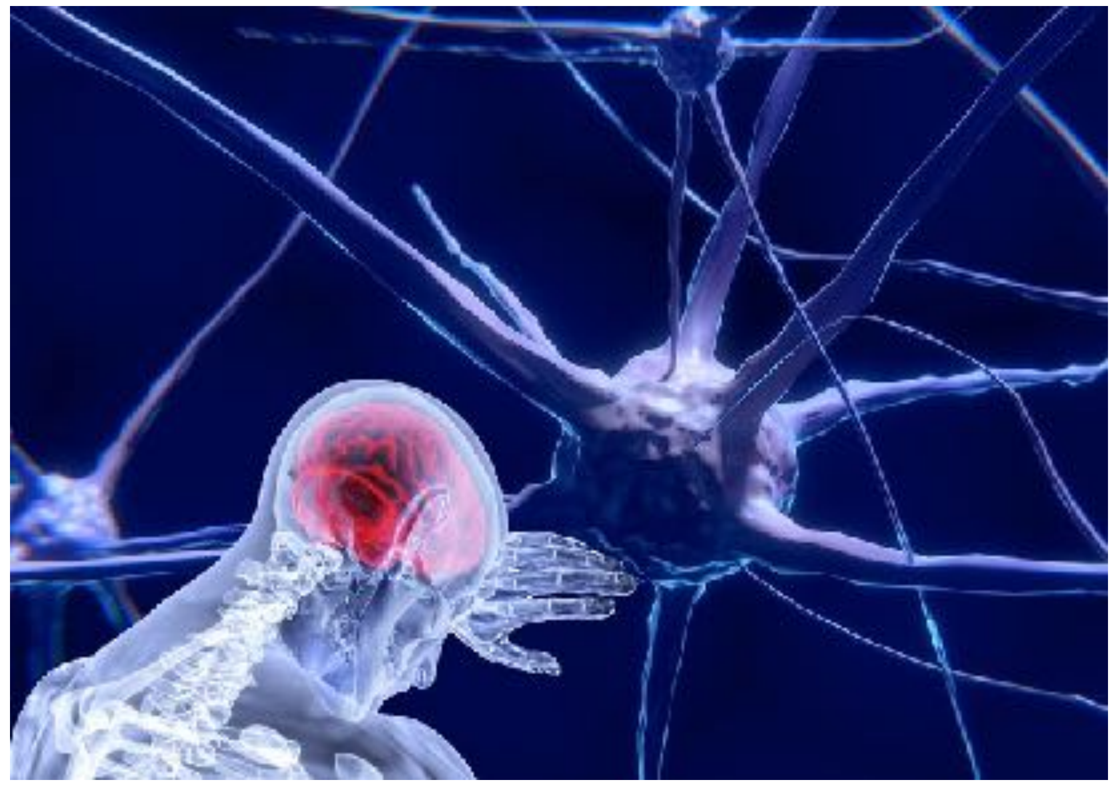
Reflection:
Turing Machine and Reality

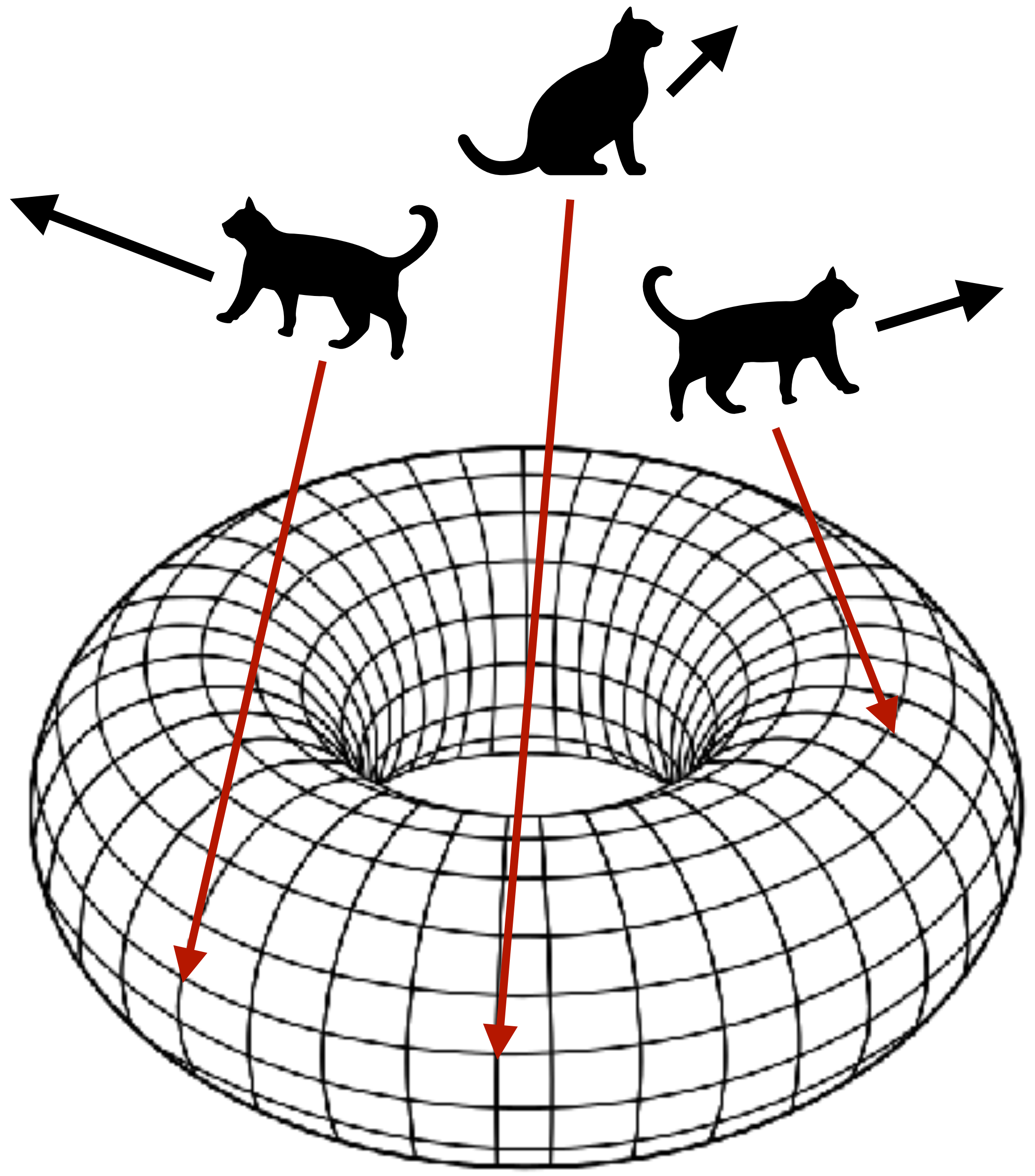
Module I: The Mathematical Foundation of Computation

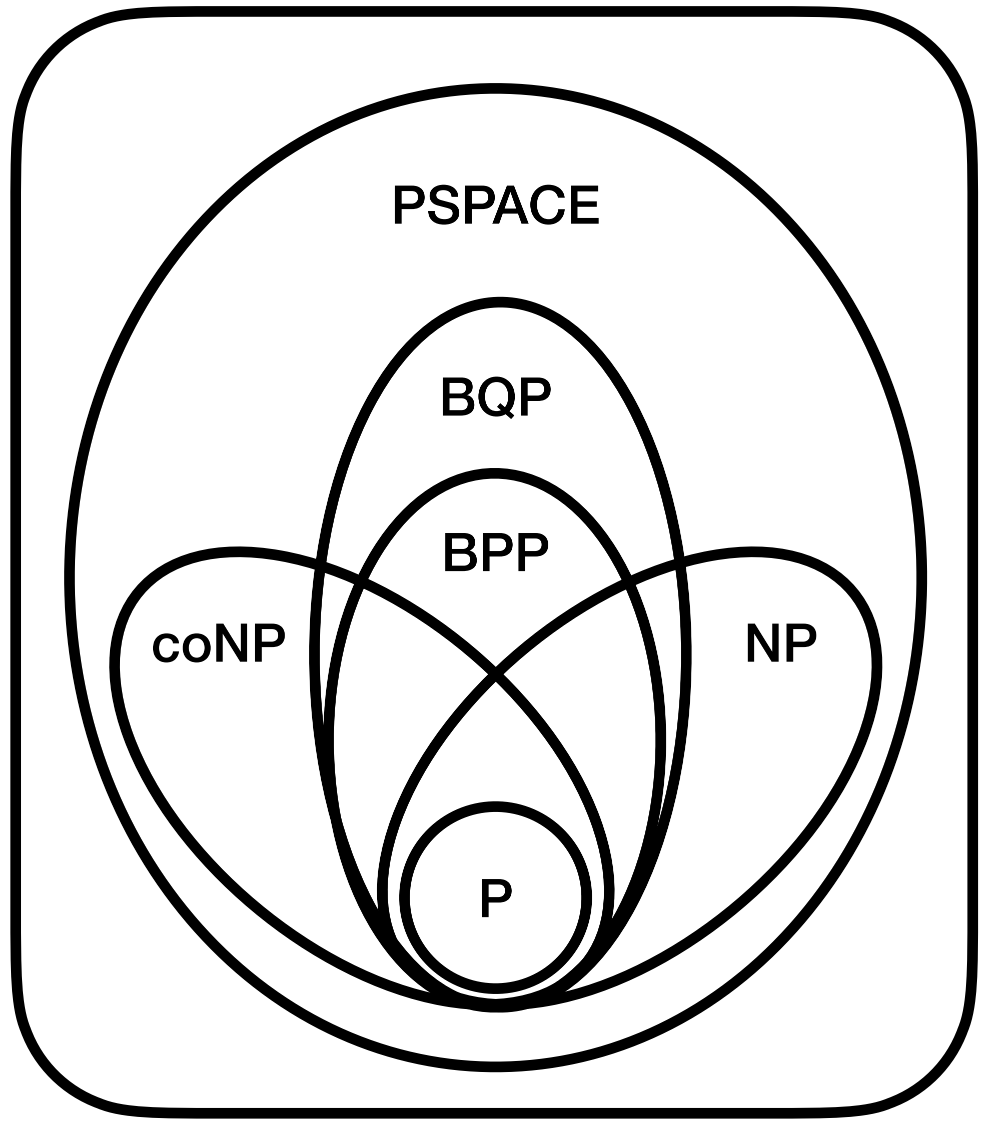
"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."

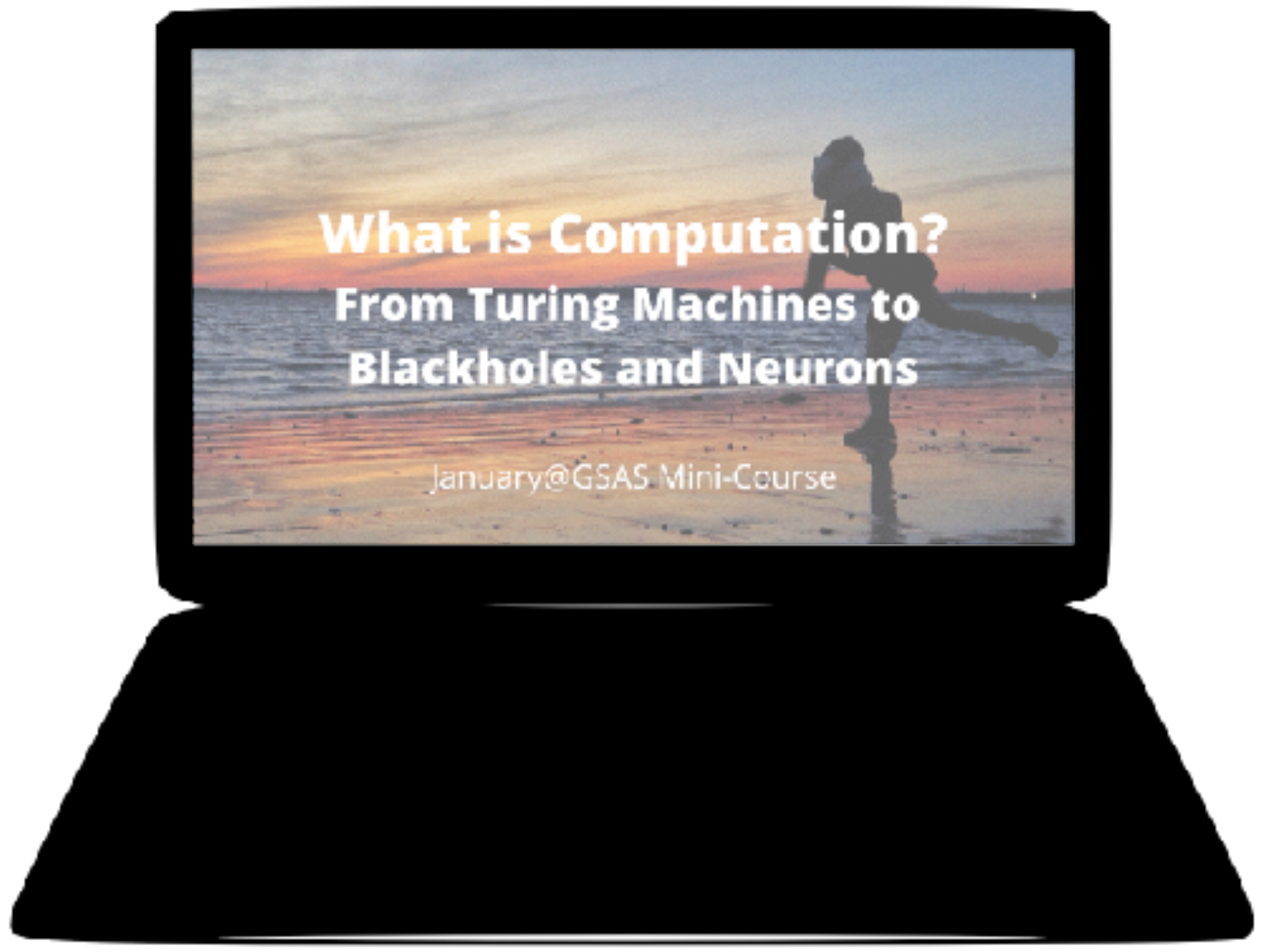
– Albert Einstein

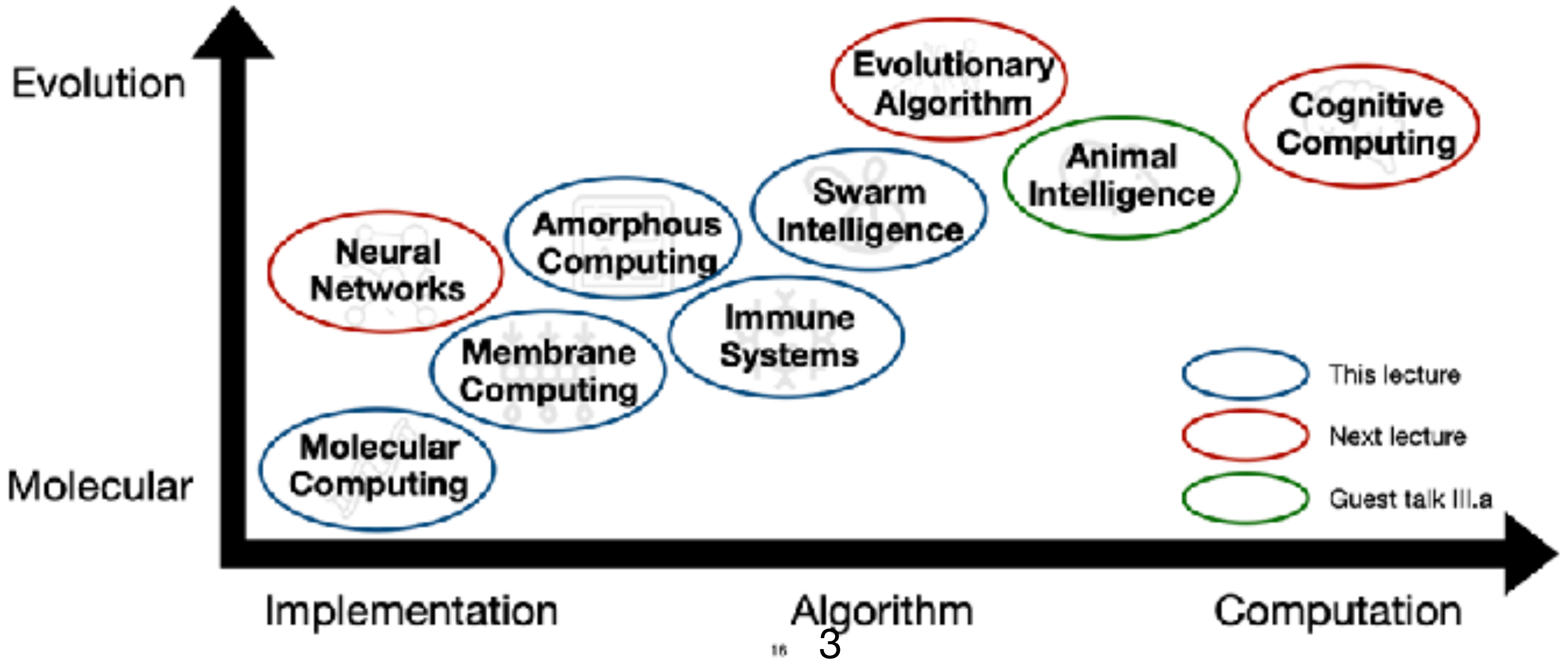


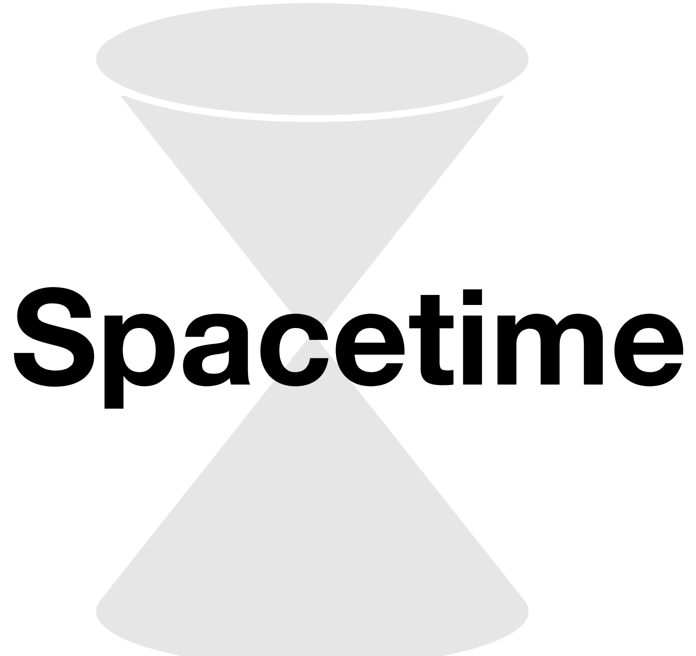












$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

So… What is Computation?

Well… I can always squeeze out an answer for you…

But, it might not be that illuminating…

Let’s Ask More Questions!?

Mathematics

Physics

Biology

Let’s Ask More Questions!?

Does mathematics characterize computation? What’s missing?

Is the physical world a computer? So what?

How can we say about computation under uncertainties and unknowns?

A Clarification

Computer Science $\subsetneq$ Computation

Philosophy of Computation

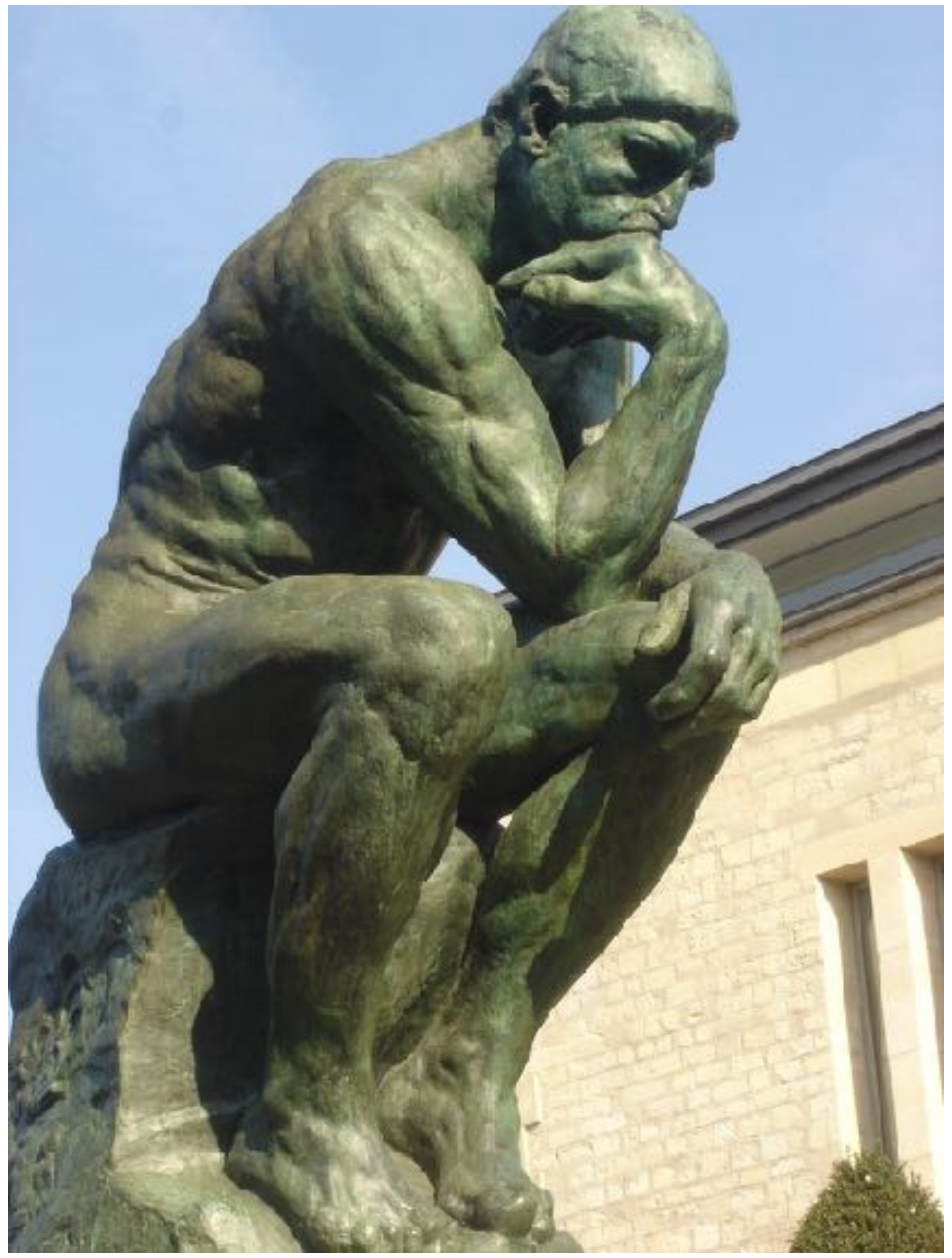
This is not the philosophy of computer science!

Q: What's the relationship between mathematics and computation?

Q: Does computation have to be logical/ mechanical/automatic?

Q: What's the relation of computation and meaning/understanding?

Q: Is everything doing some kind of computation (i.e., pancomputationalism)!?

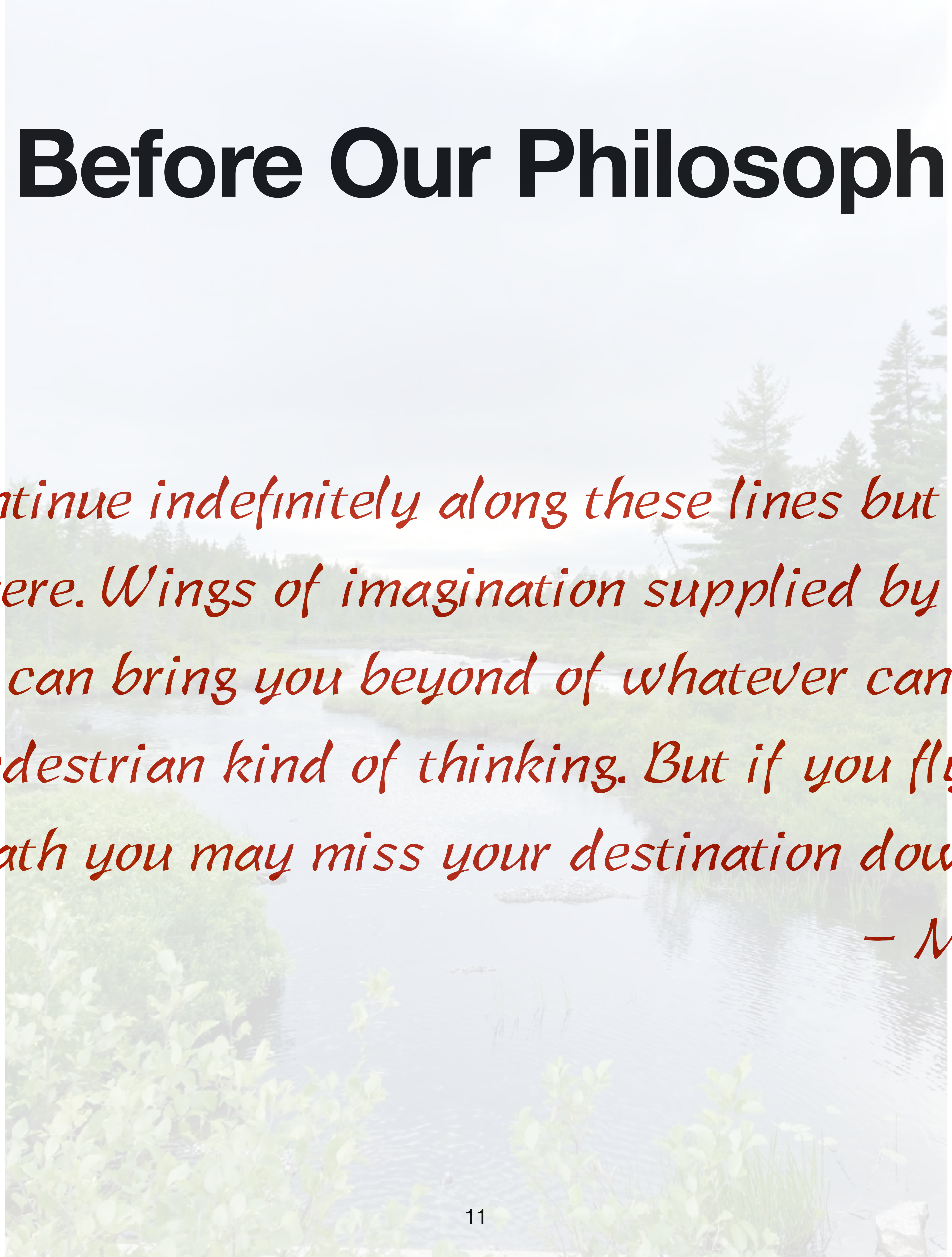


Computo, ergo sum?

I compute, therefore I am?

*Davis, Philip J. "Toward a Philosophy of Computation." *For the Learning of Mathematics* 3.1 (1982): 2-5.

A Reminder Before Our Philosophical Journey

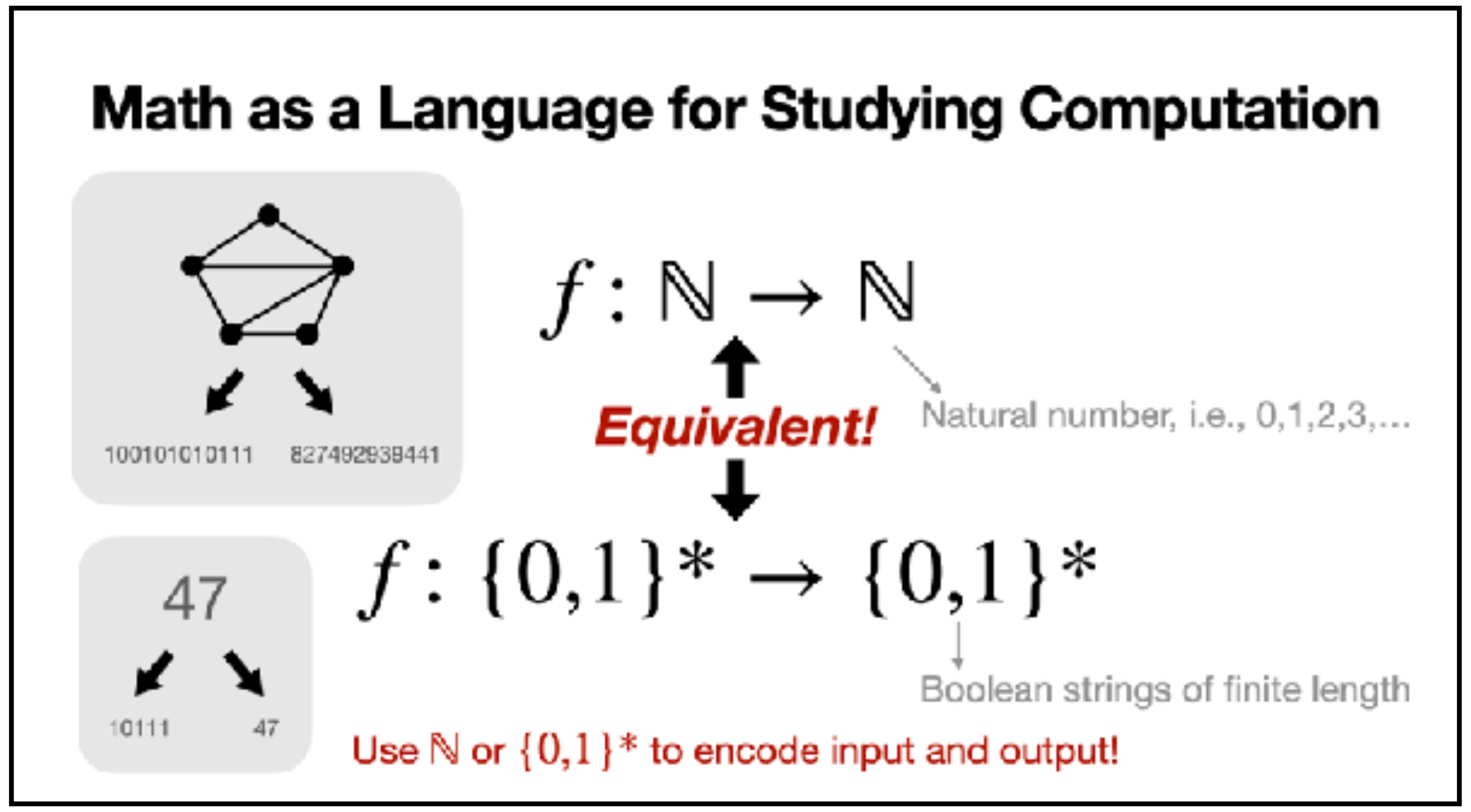


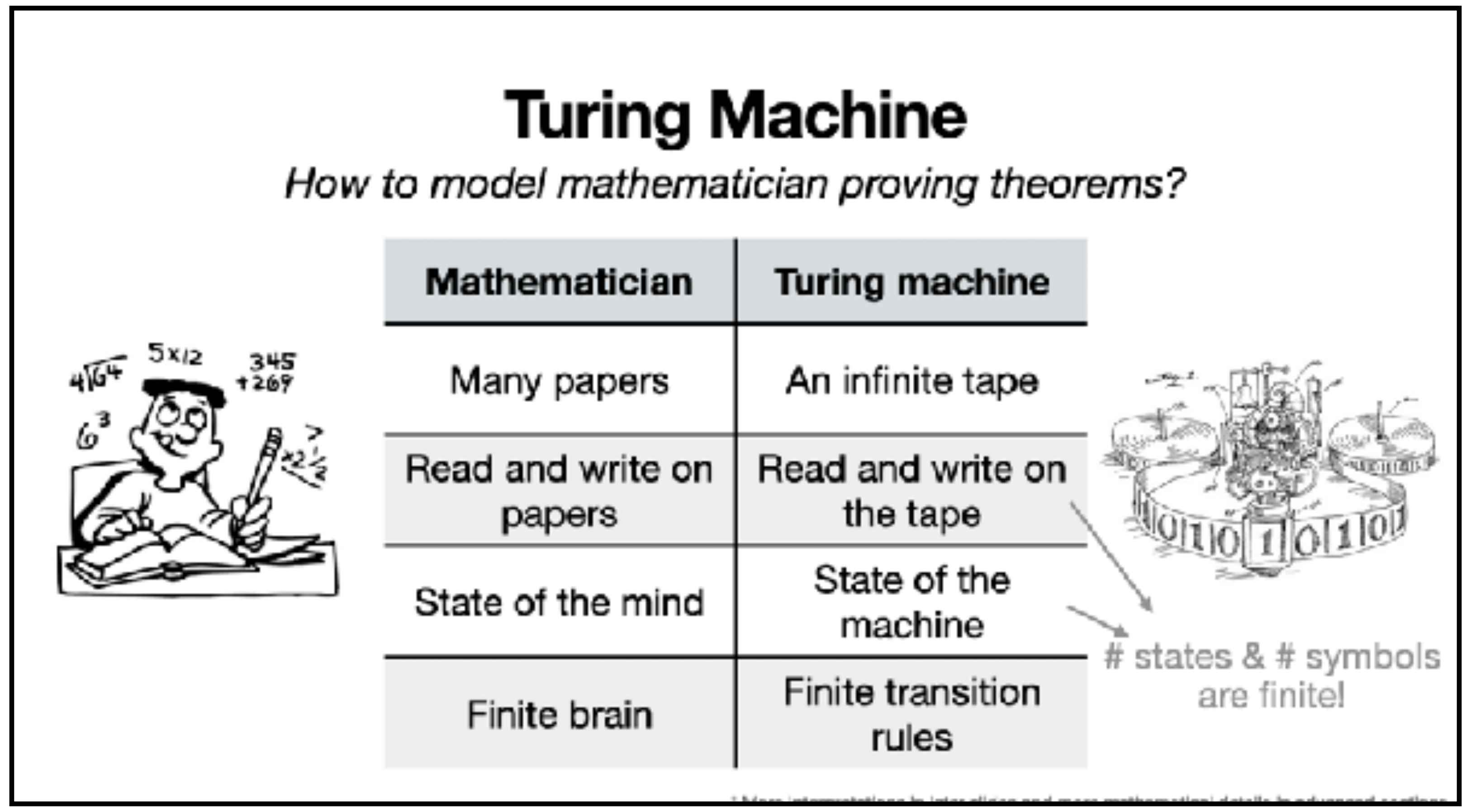
"One may continue indefinitely along these lines but one has to stop somewhere. Wings of imagination supplied by the power of mathematics can bring you beyond of whatever can be reached by a more pedestrian kind of thinking. But if you fly too high in the sky of math you may miss your destination down on Earth."

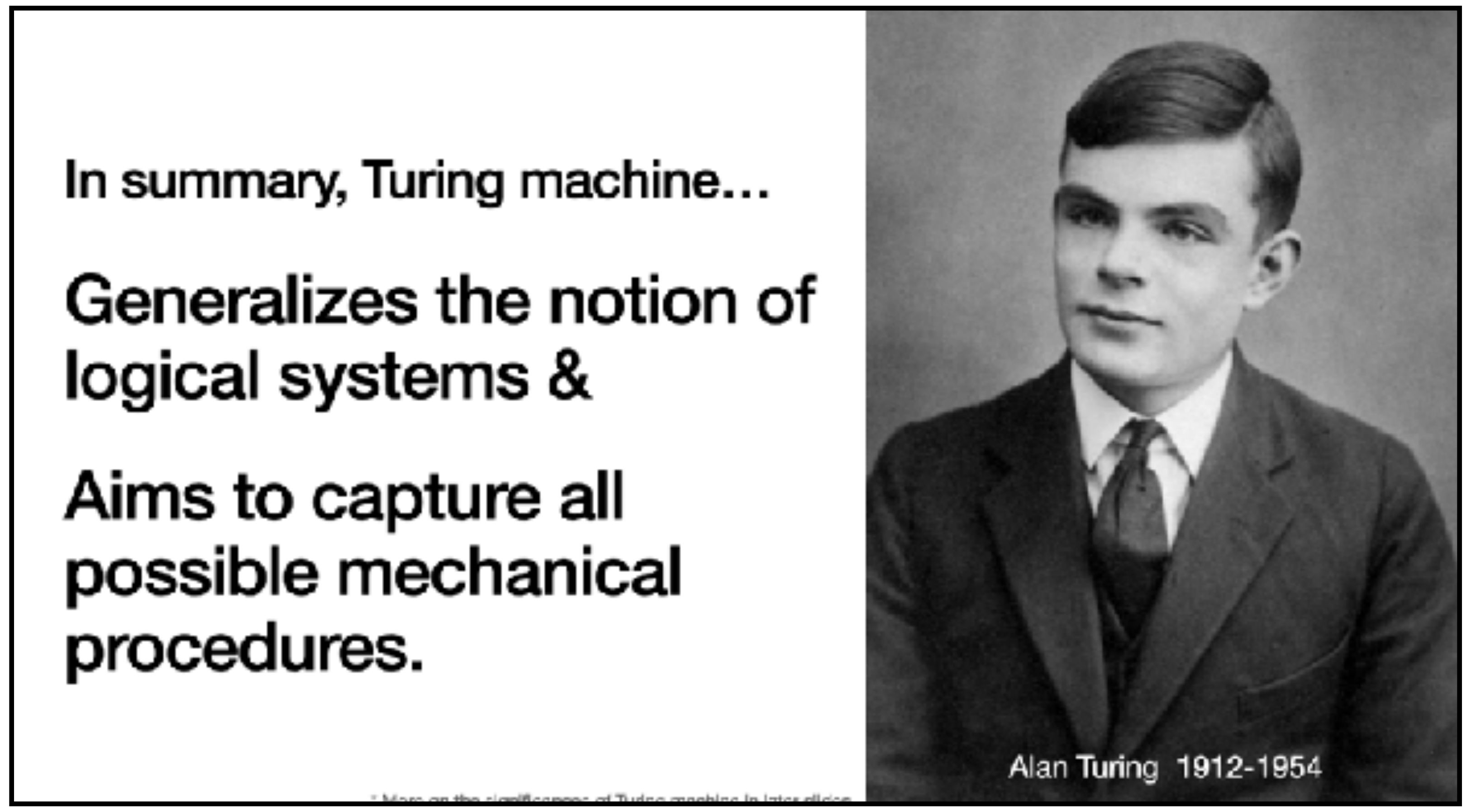
– Misha Gromov

Back to Lecture I.a & I.b

Turing Machine and Why?

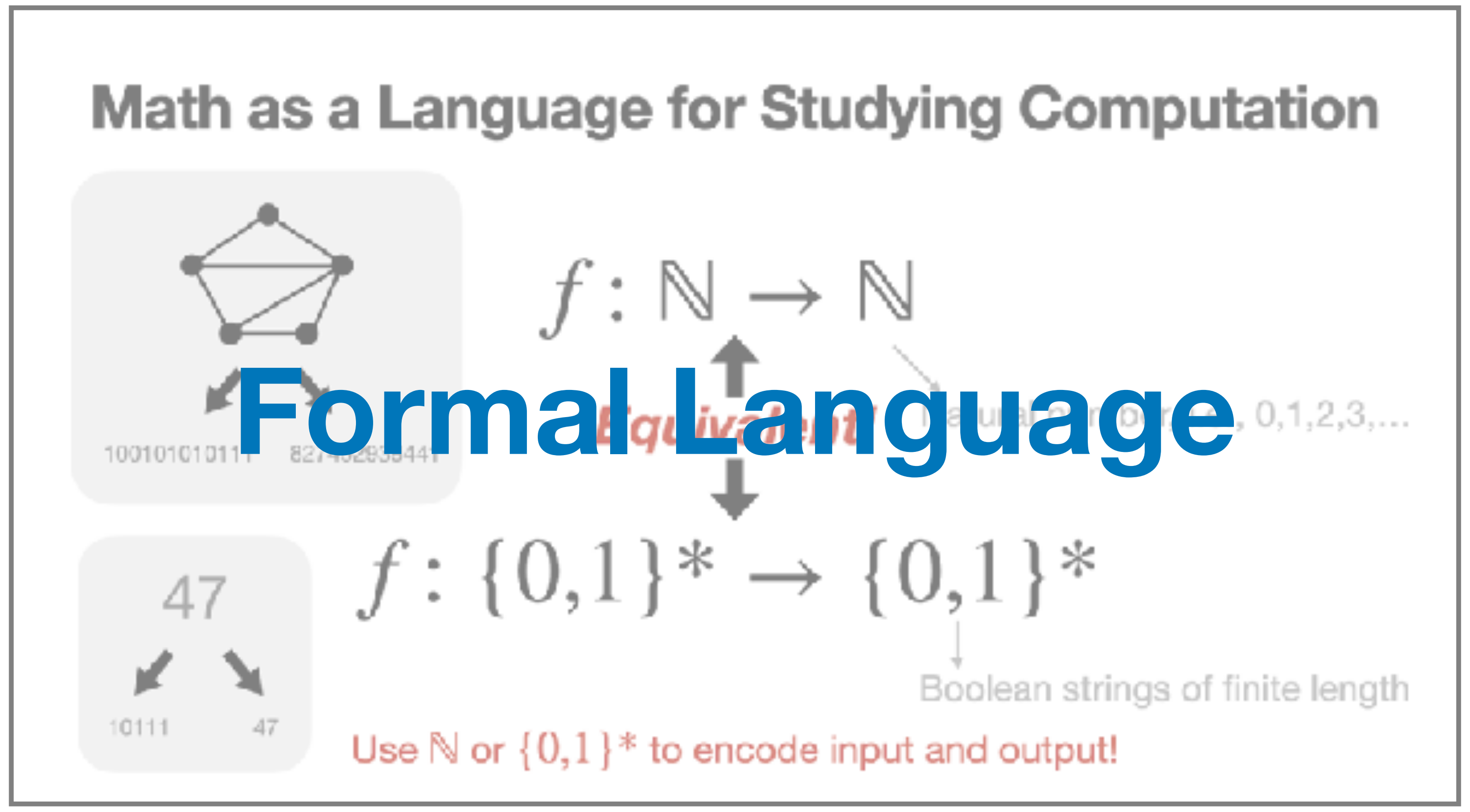


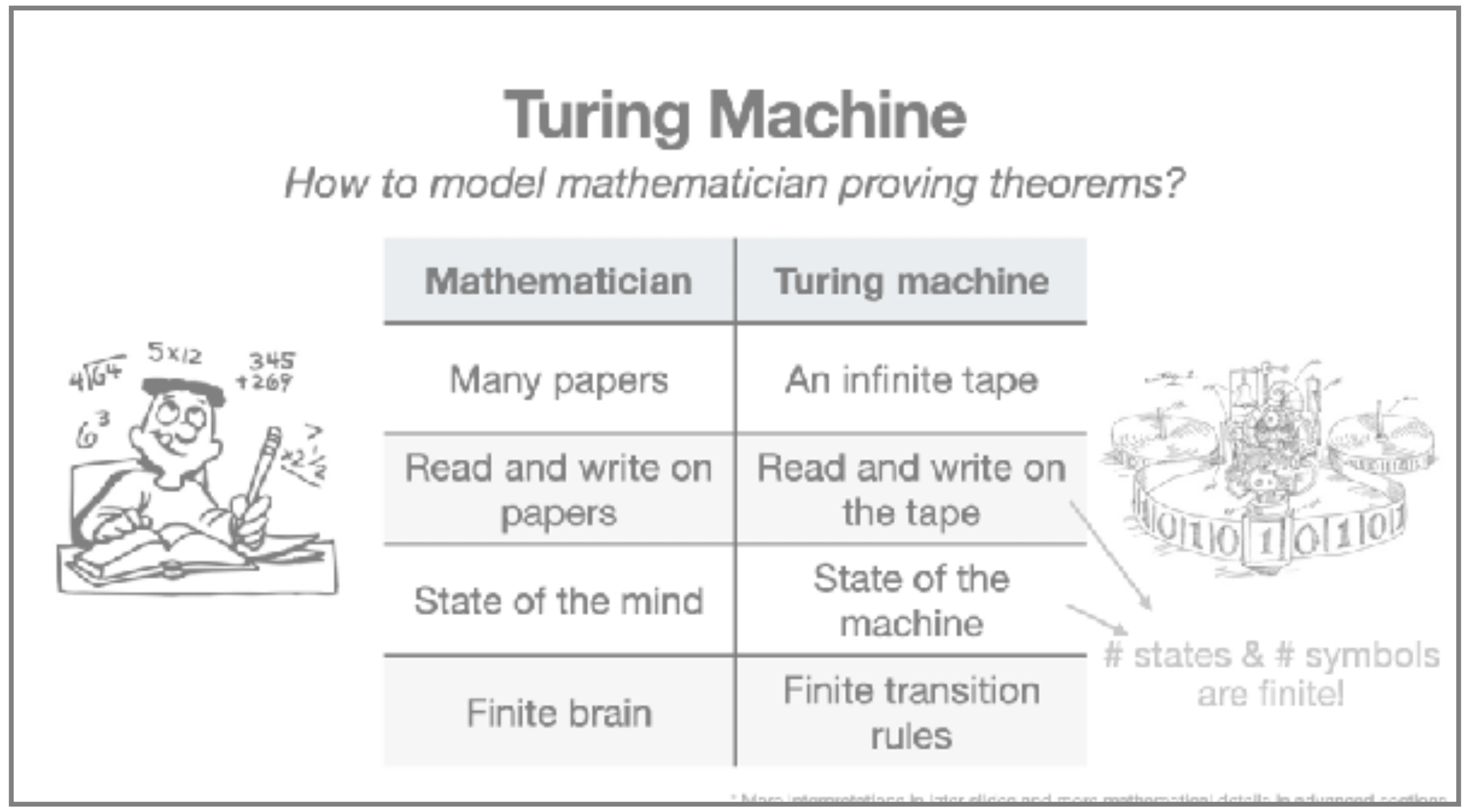


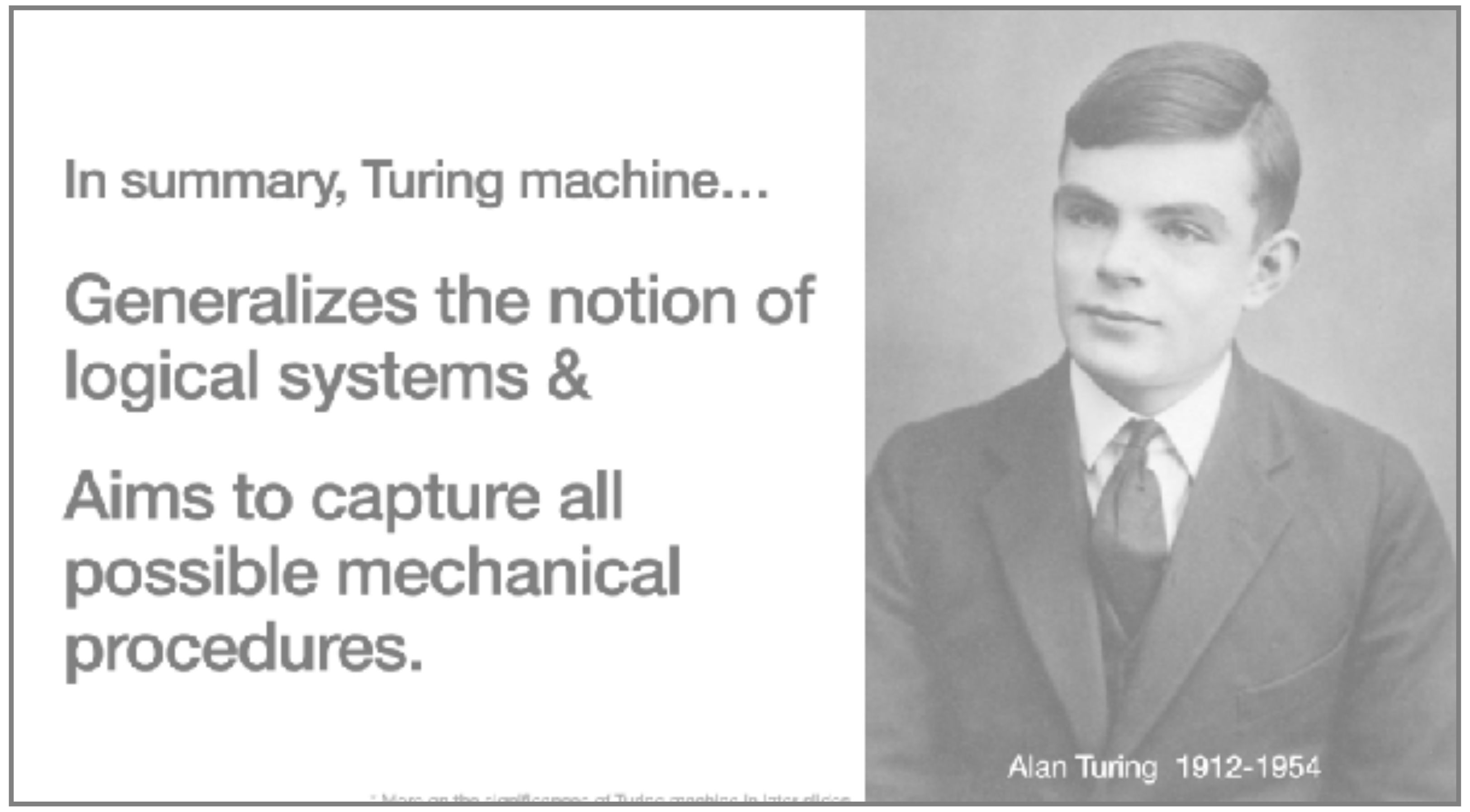


Church-Turing Thesis

All realizable computation in the physical world can be done by a Turing machine.







Turing Machine and Why?

Q1: Does the computation we experienced in reality be captured by the math formulation we discussed (i.e., formal language)?
Also, does Turing machine capture all possible mechanical procedures? If not, why?

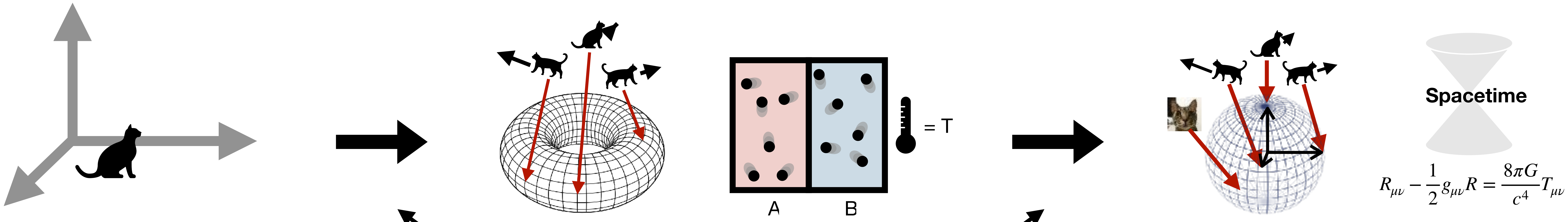
Beyond formal language and Turing machine?

Q2: Currently, the theory of computation seems to be a pure subfield of mathematics. How can we provide more insights in the relation to reality?

Computation and reality!?

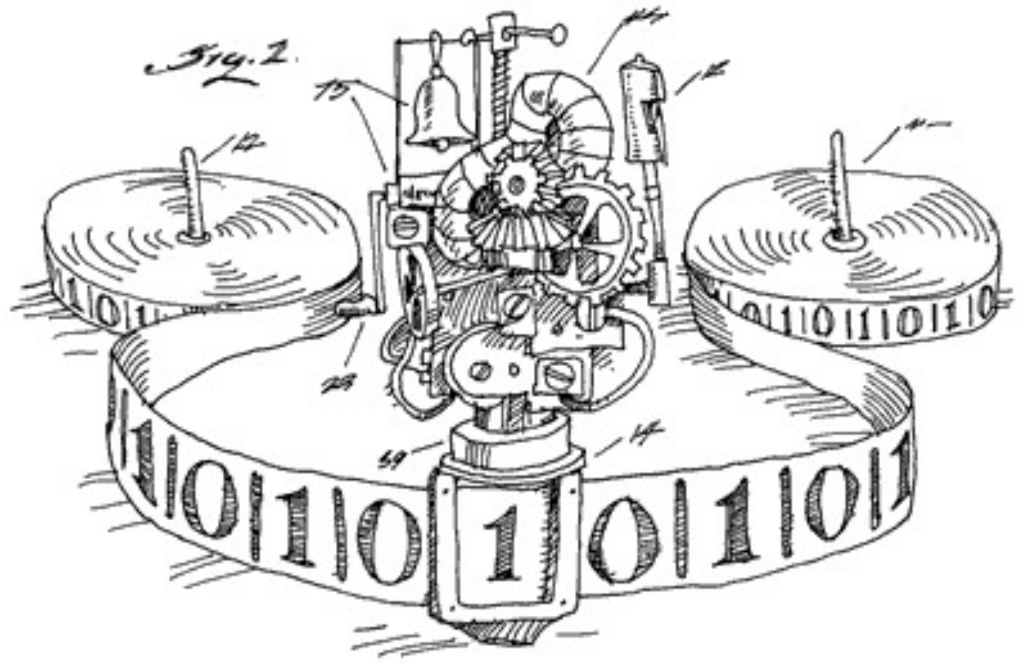
An Analogy the Theoretical Physics

Through the lens of philosophy of science



Thomas Khun
1922-1996

Paradigm Shift



We are living in the paradigm of Turing machine!?

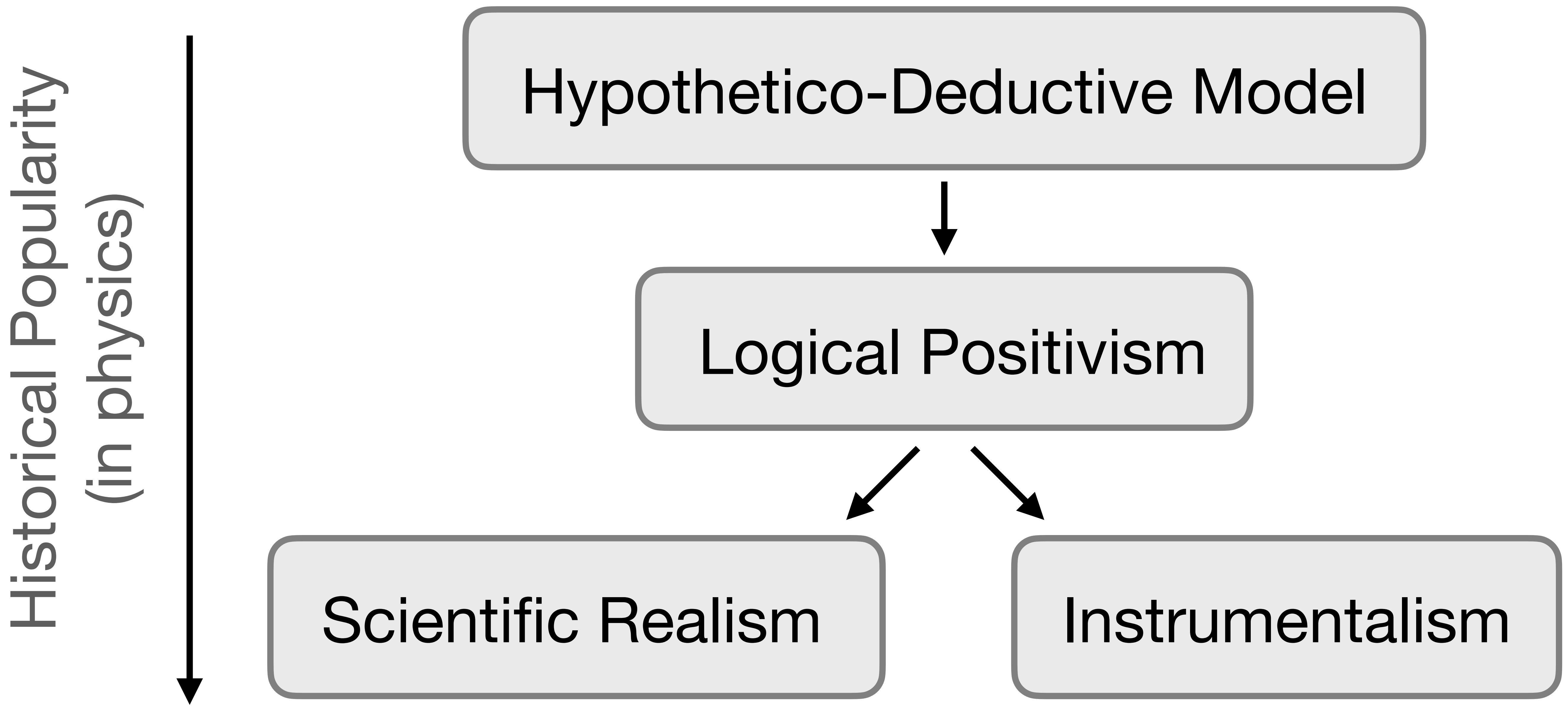
A Bird-Eye View on Philosophy of Science

In quest of a philosophy of computation?

Q: What qualifies as science?

Q: How reliable are scientific theories?

Q: What's the ultimate purpose of science?



A (Biased) Crash Course on Philosophy of Science

Hypothetico-Deductive Model

- Build hypothesis and do deduction!
- E.g., some sub-fields in applied physics and biology?

Logical Positivism

- Build on top of hypothetico-deductive model.
- Emphasize the isolation of “theoretical term” and “observational term”.
- Accept the gap between “analytic statements” and “synthetic statements”.
- Dream of the “unity of science”.
- E.g., math, TCS, string theory?

Scientific Realism

- Having belief in the existence of “ideal theory”.
  - There are several “criteria” for what can be called an ideal theory.
- The scientific goal is to approach an ideal theory and produce true descriptions of things in the world.

Instrumentalism/Pragmatism

- The worth of an idea is based on how effective it is in explaining and predicting phenomena.
- Rejecting scientific realism, and sometimes known as antirealism/nonrealism.

Note that a scientist could have different philosophy for different (sub-)fields!

A Bird-Eye View on Philosophy of Science

In quest of a philosophy of computation?

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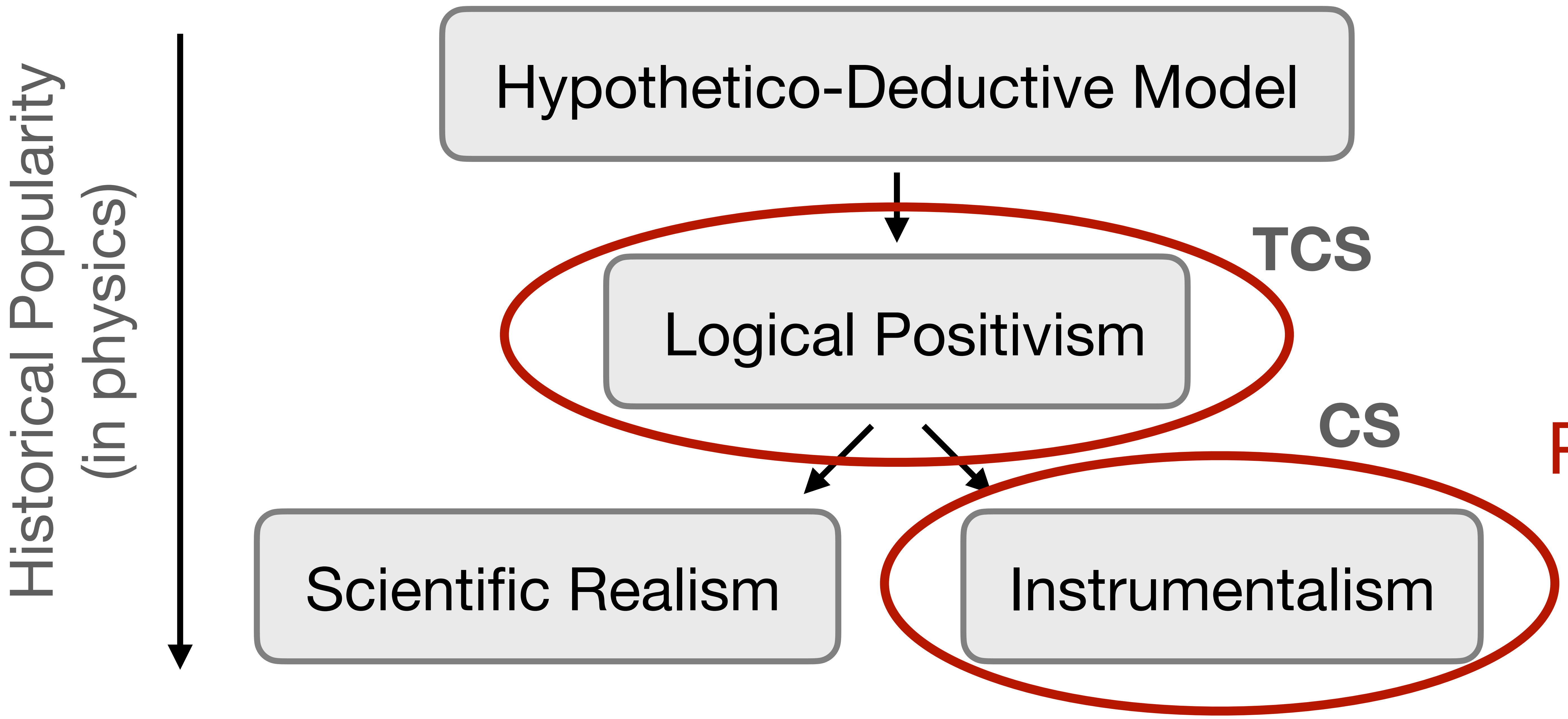


There are much more branches! E.g., constructive empiricism, deductive-nomological model, emergentism, etc.

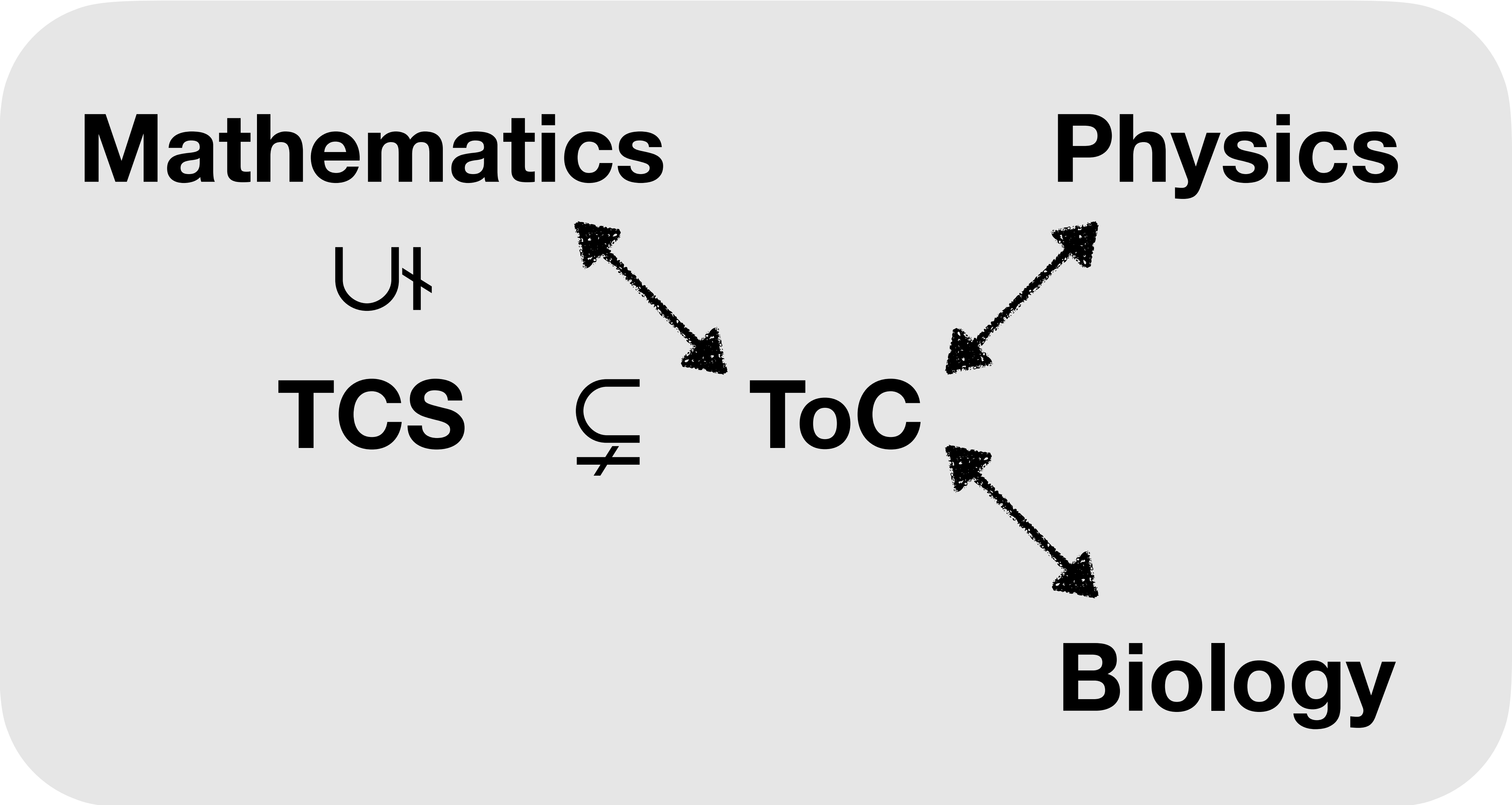
Provide frameworks for us to think about the connection to reality!

My Personal View of Q1 and Q2

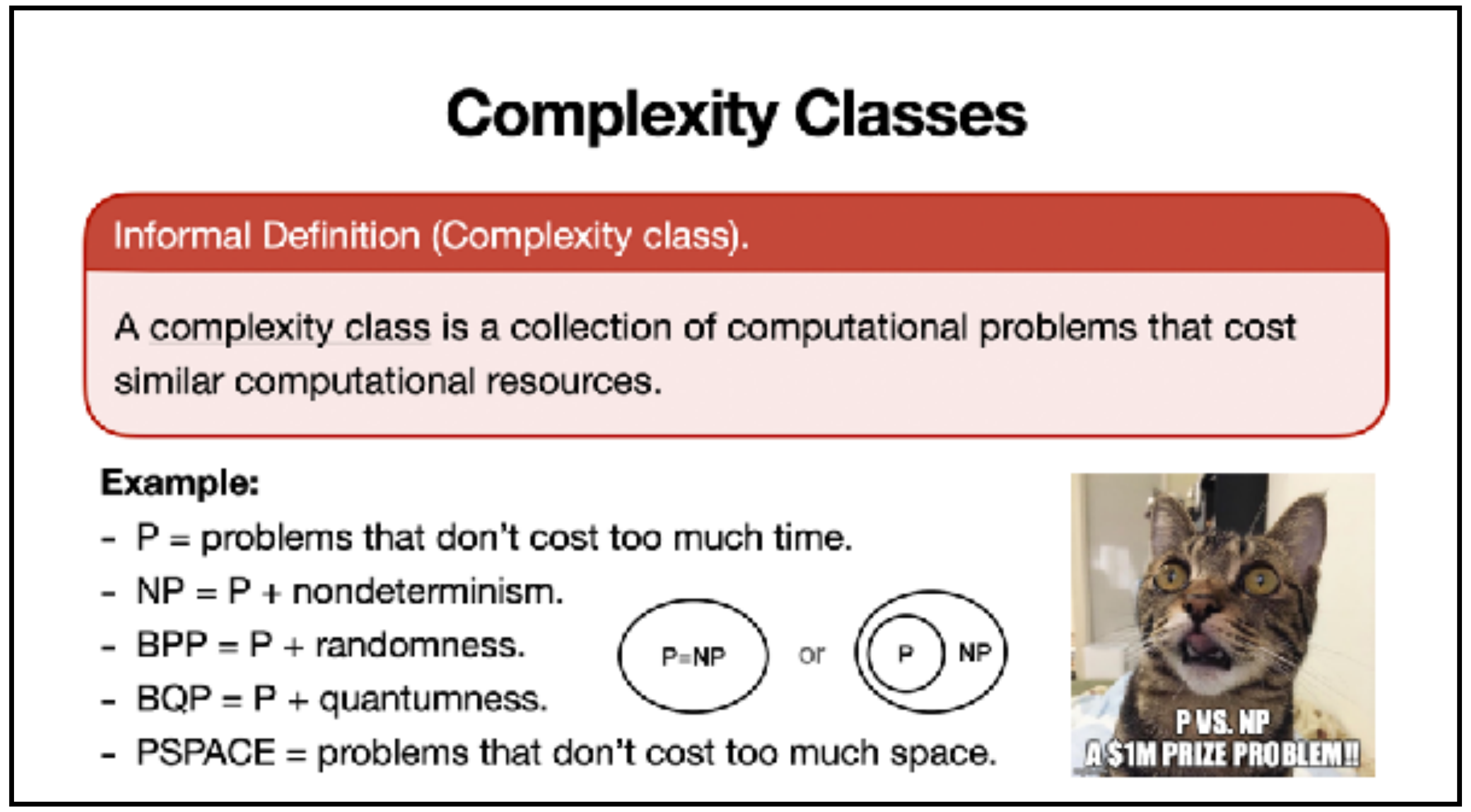
Q1: Does the computation we experienced in reality be captured by the math formulation we discussed? Also, does Turing machine capture all possible mechanical procedures? If not, why?

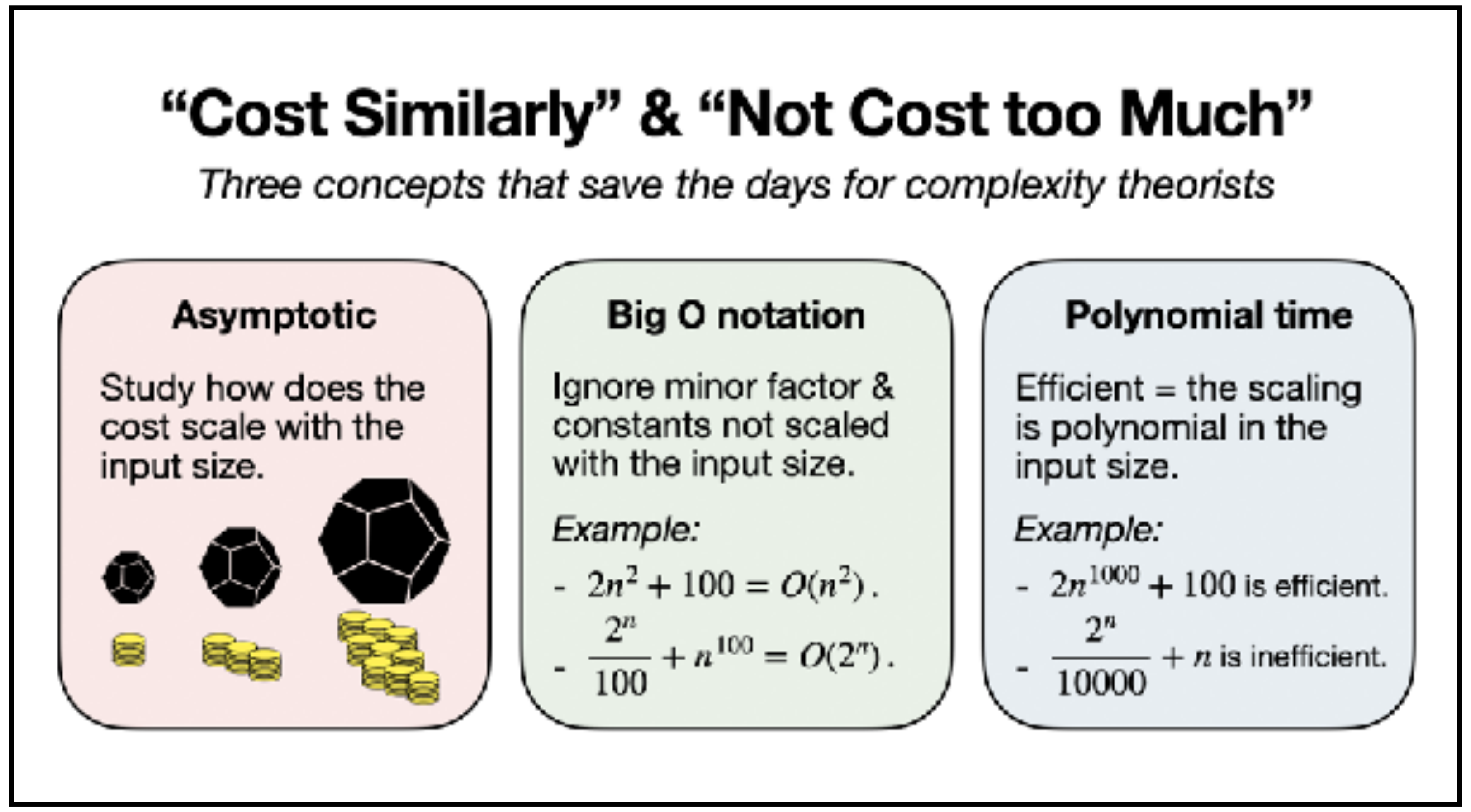
Q2: Currently, the theory of computation seems to be a pure subfield of mathematics. How can we provide more insights in the relation to reality?

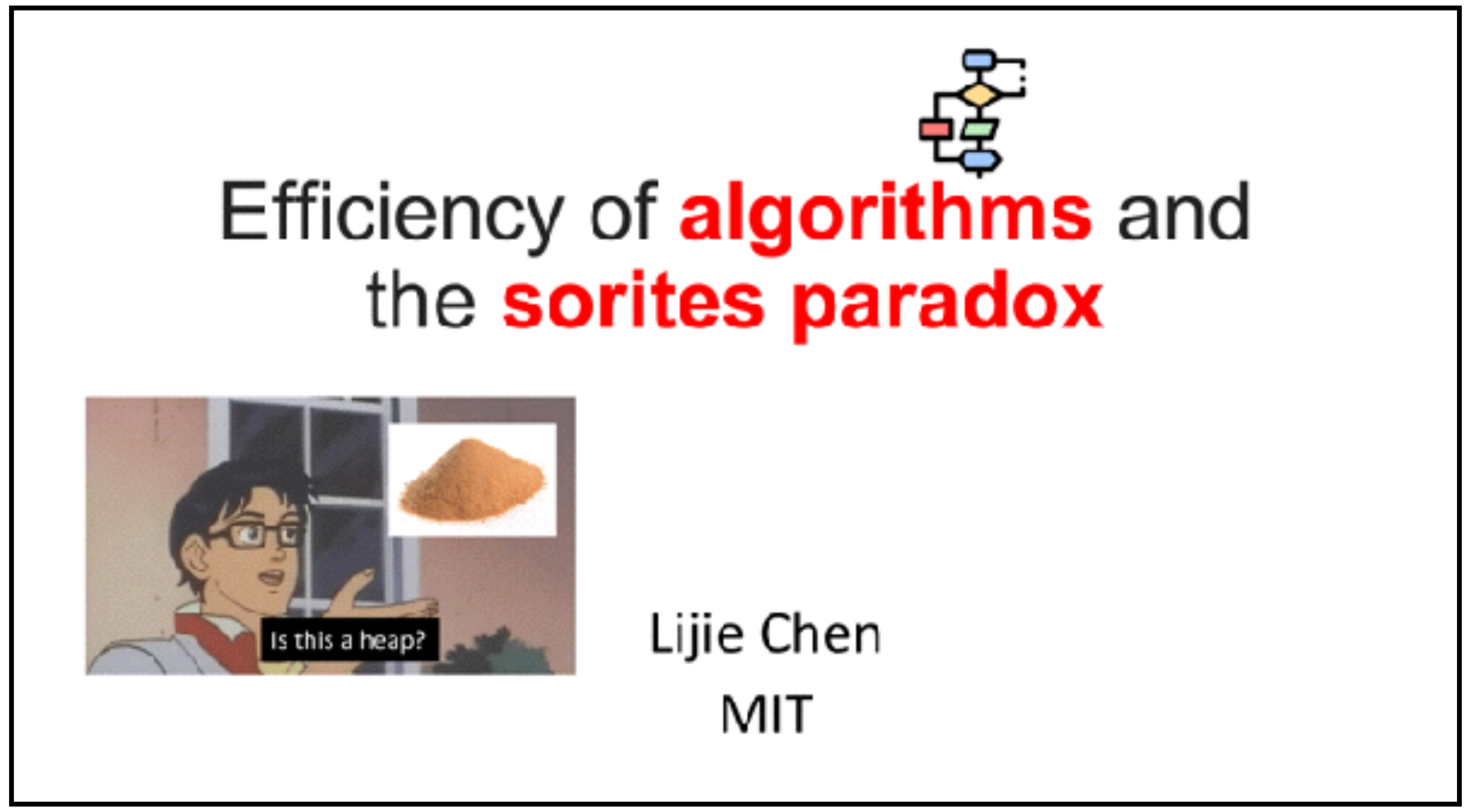
- I think the long-term goals of Theoretical CS (TCS) still have its intellectual merits.
- But, we should be open-minded to a potential new Theory of Computation (ToC) which aims to touch reality.

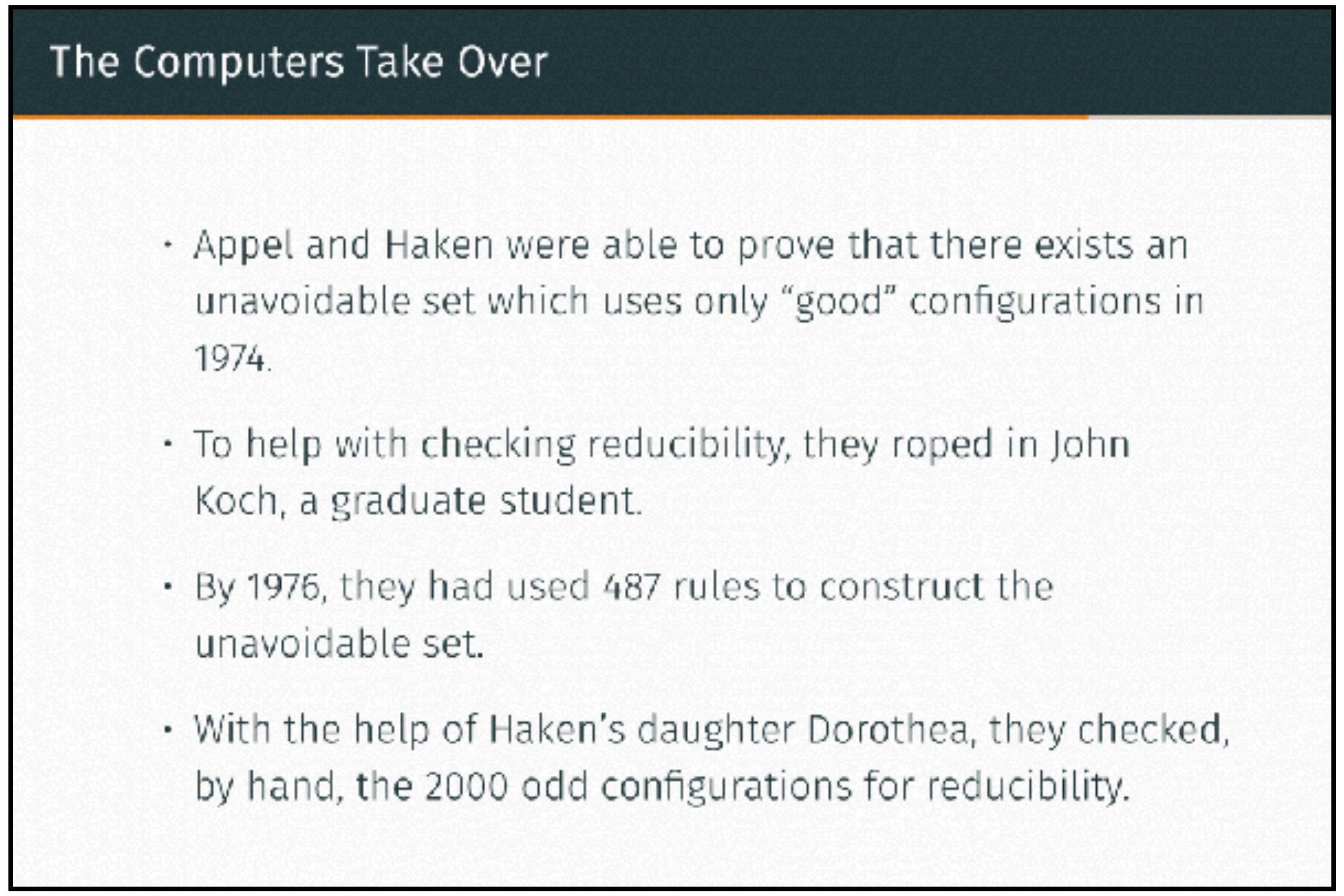


What Does TCS Teach Us?









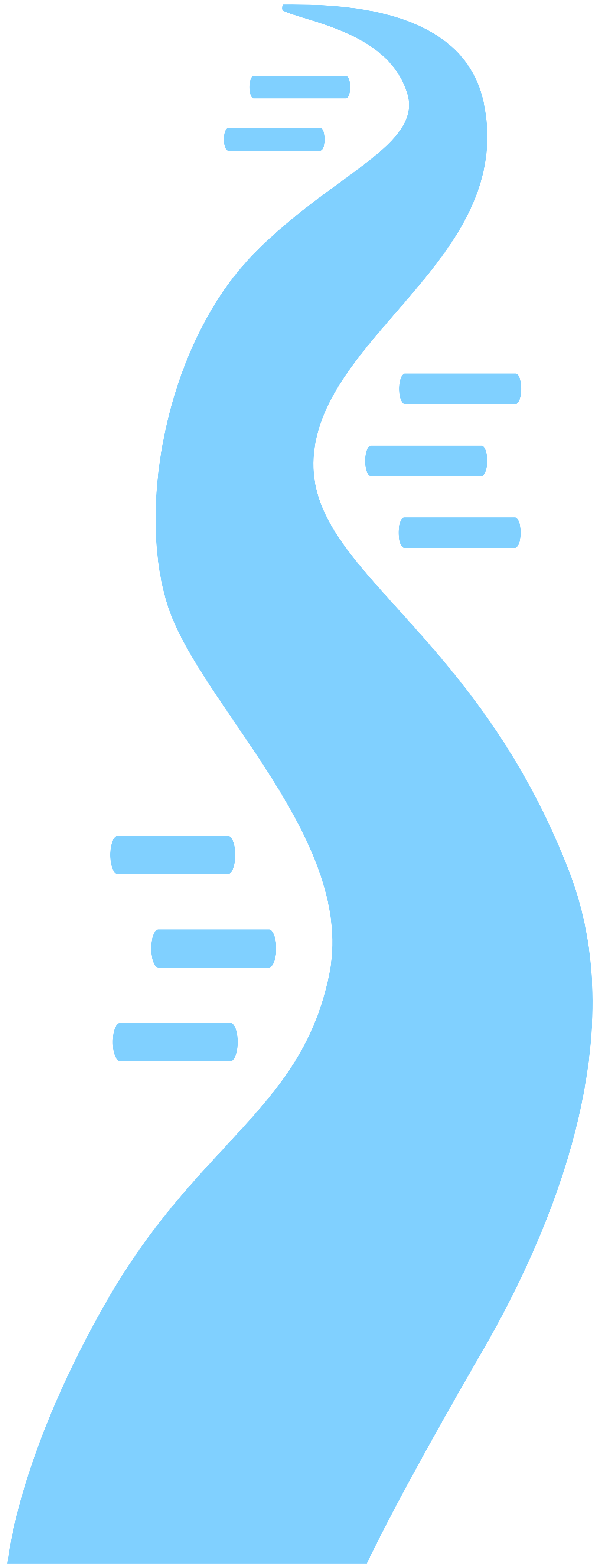
Q3: How to interpret worst-case and asymptotic analysis in reality?

Q4: What's the relation between computation and understanding? E.g., what do we mean by understanding a computation? Is mathematical proof necessary? What if the proof is complicated?

Meaningful Inputs?

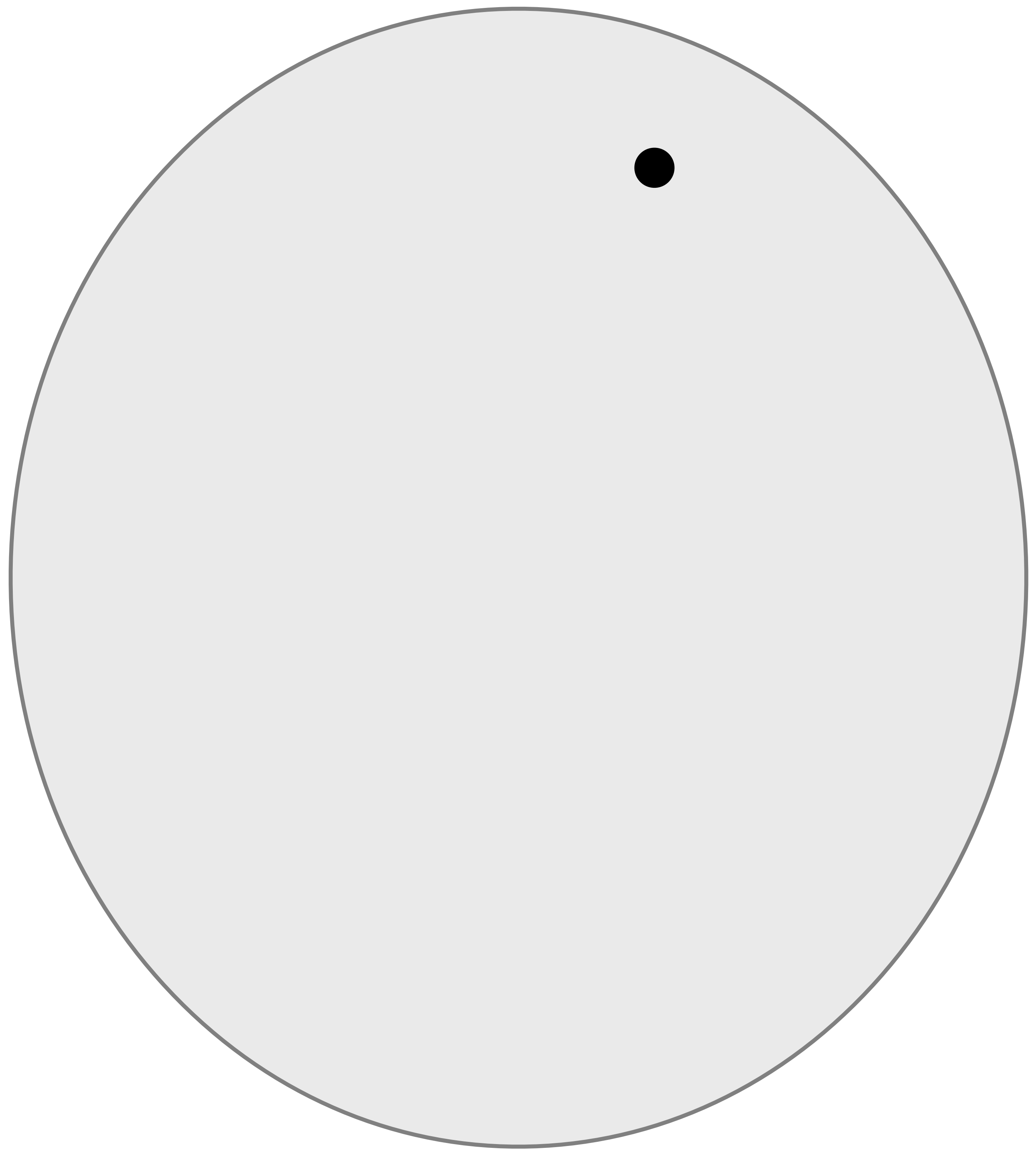
Mathematical World

- Input assumption:
  - Worst-case analysis.
  - Average-case analysis.
  - Smooth analysis.
- Asymptotic analysis.

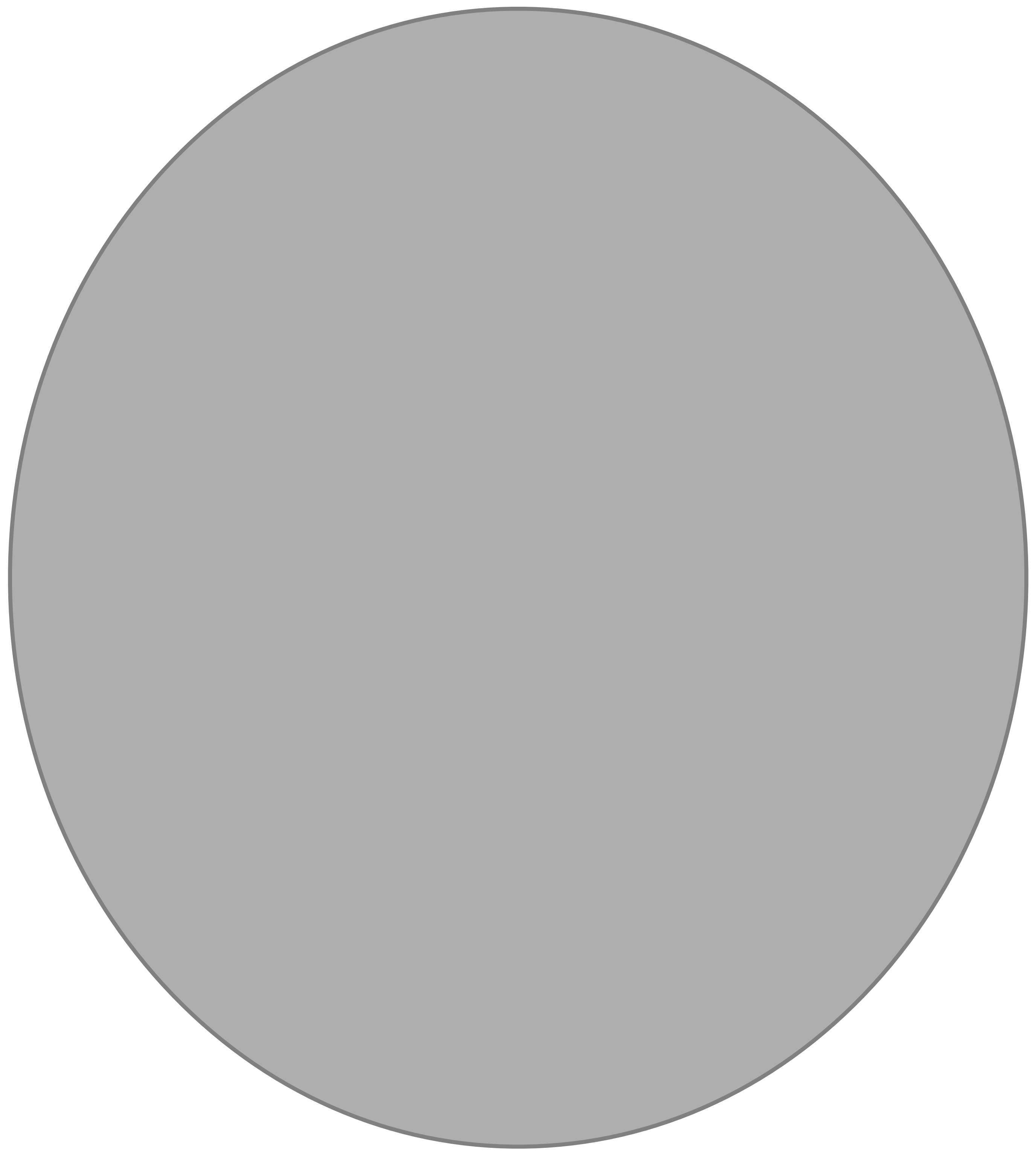


Reality

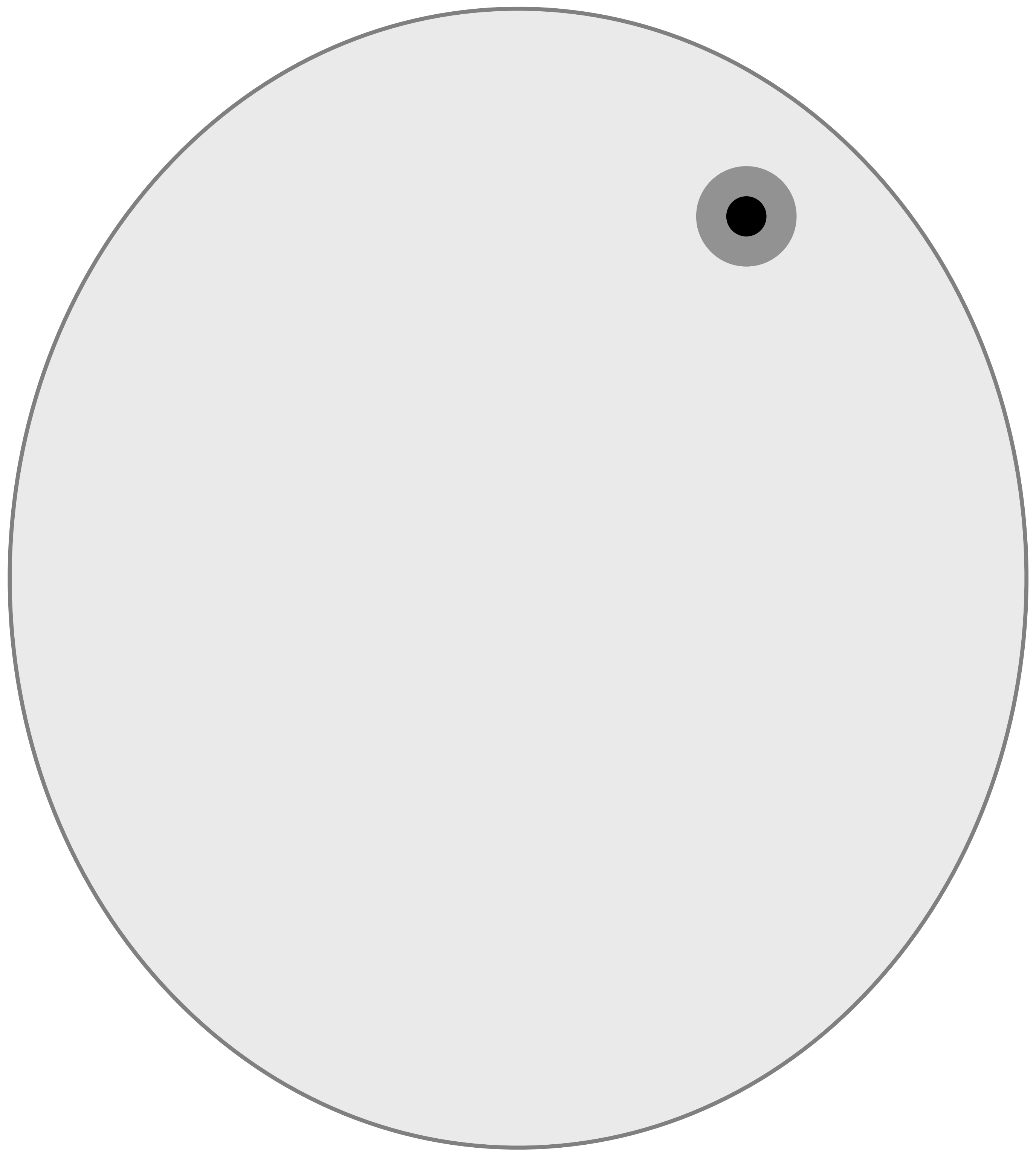
Different Input Assumptions



Worst-case analysis



Average-case analysis



Smooth Analysis

Meaningful Inputs?

Mathematical World

- Input assumption:
  - ✦ Worst-case analysis.
  - ✦ Average-case analysis.
  - ✦ Smooth analysis.
- Asymptotic analysis.

Reality

Q: How to model empirical inputs?

Q: How to handle finite problem?

My view/proposal:

- Independently study “what are meaningful inputs”.
- Learn from physics, biology, and emergence.

Understanding & Computation

Q4: What's the relation between computation and understanding? E.g., what do we mean by understanding a computation? Is mathematical proof necessary? What if the proof is complicated?

Mathematical proof

Heuristics

Empirical performance

I think we need all of them in the theoretical study of computation, however, how to bridge them? Again, physics could be a good example to learn from!

Summary

Questions of This Lecture

Q1: Does the computation we experienced in reality be captured by the math formulation we discussed (i.e., formal language)? Also, does Turing machine capture all possible mechanical procedures? If not, why?

Q2: Currently, the theory of computation seems to be a pure subfield of mathematics. How can we provide more insights in the relation to reality?

Q3: How to interpret worst-case and asymptotic analysis in reality?

Q4: What's the relation between computation and understanding? E.g., what do we mean by understanding a computation? Is mathematical proof necessary? What if the proof is complicated?

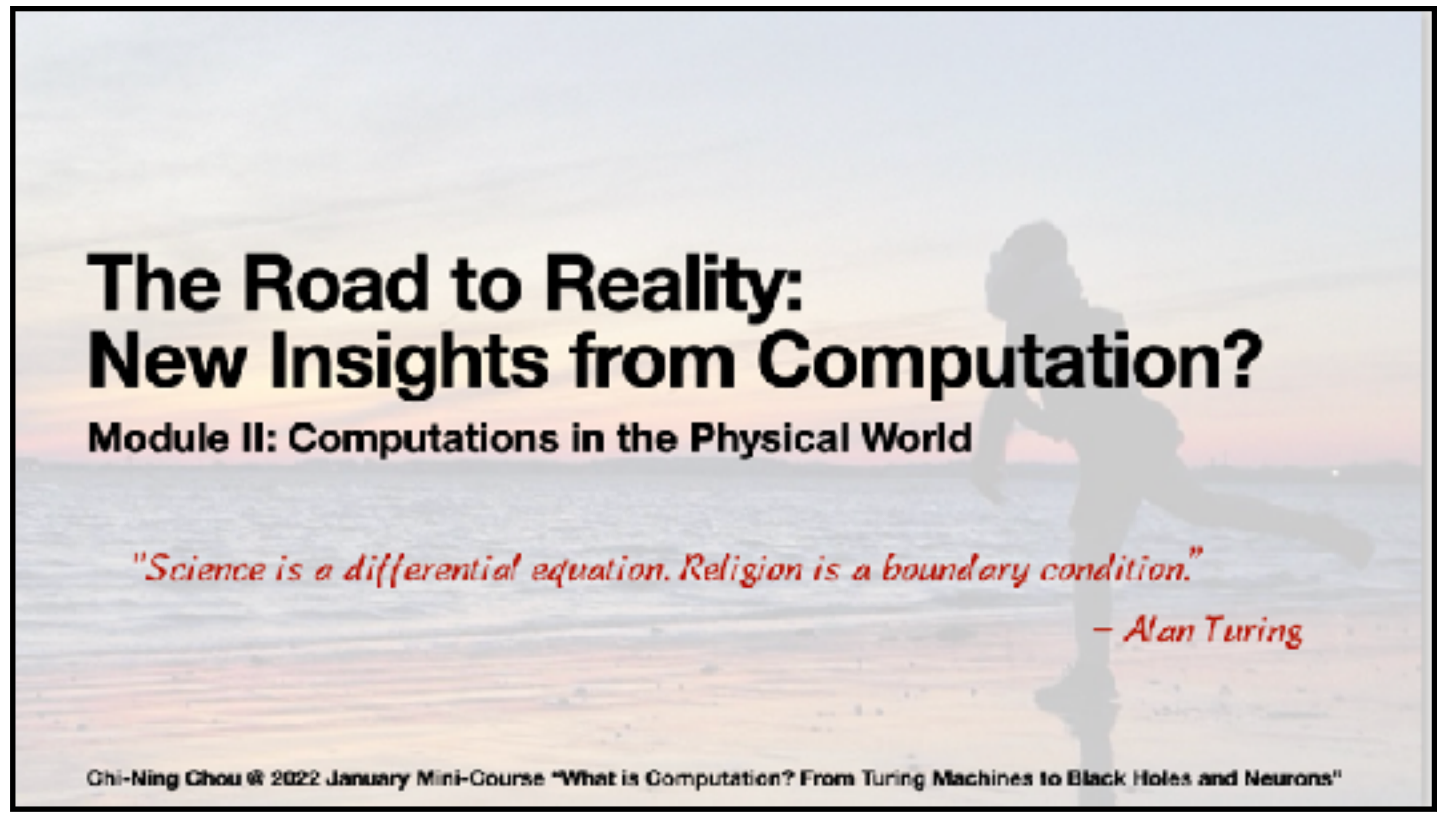
What’s Next?

Sowmya
(Jan. 18
10am-11am ET)

“Basic of Quantum Computing and Future Direction”

Avantika
(Jan. 18
11am-12pm ET)

“Quantum Complexity Theory”



Lecture II.c
(Jan. 19 10am-10:50am ET)

- Pancomputationalism
- Physical Church-Turing Thesis
- Physical systems and algorithms
- MIP*=RE

“A Road to Totality: Between Art and Computation”

Zhiqian Wang
(Jan. 19 11am-12pm ET)

Salvador
(Jan. 19
2pm-3pm ET)

“DNA Computing, Cellular Automata, and Beyond”

References

Articles:

- Davis, Philip J. "Toward a Philosophy of Computation." *For the Learning of Mathematics* 3.1 (1982): 2-5.
- Knuth, Donald E. "Computer science and its relation to mathematics." *The American Mathematical Monthly* 81.4 (1974): 323-343.

Introductory Books:

- Moore, Cristopher, and Stephan Mertens. The nature of computation. OUP Oxford, 2011, link.
- Barak, Boaz. Introduction to Theoretical Computer Science. Online, link.
- Rautenberg, Wolfgang. A concise introduction to mathematical logic. New York, NY: Springer, 2006, link.

Advanced Books:

- Wigderson, Avi. Mathematics and computation. Princeton University Press, 2019, link.
- Arora, Sanjeev, and Boaz Barak. Computational complexity: a modern approach. Cambridge University Press, 2009, link.

Fun reads:

- Aaronson, Scott. “Why philosophers should care about computational complexity.” Computability: Turing, Gödel, Church, and Beyond 261 (2013): 327, link.

* Many icons in the slides were made by Freepik from www.flaticon.com