(Nearly) Efficient Algorithms for the Graph Matching Problem on Correlated Random Graphs

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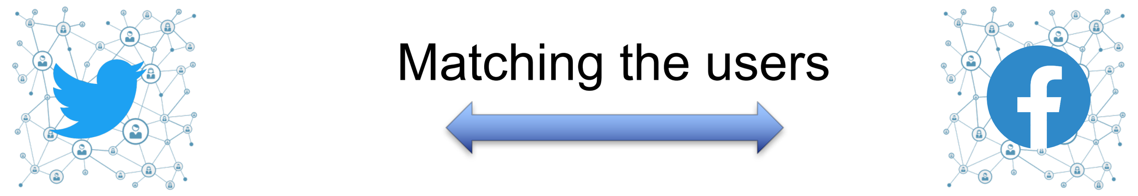
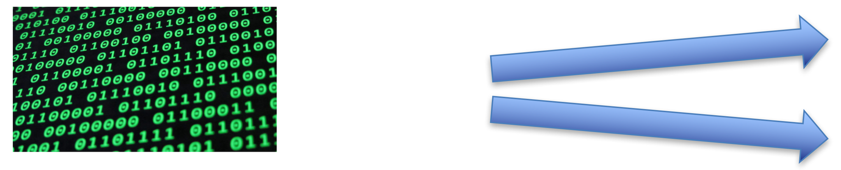
❑ Motivation

➢ De-anonymization (e.g., matching social networks)



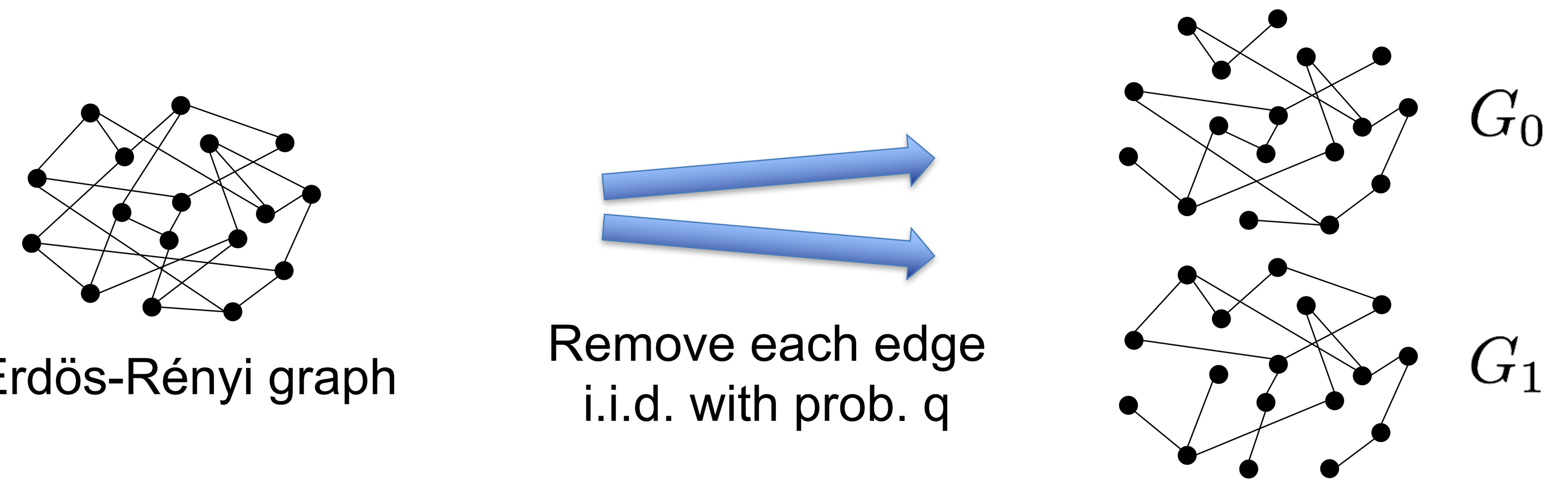
➢ Malware detection (e.g., finding suspicious patterns in a code)



❑ Problem Formulation

➢ The distance between two graphs: $\min_{\pi \in \mathcal{S}_n} \|G_0 - \pi(G_1)\|_0$.

➢ Input model: **Correlated Erdös-Rényi Graphs**.



➢ Two computational problems:

- **Graph similarity: hypothesis testing.** Given (G_0, G_1), distinguish (i) correlated Erdös-Rényi and (ii) independent Erdös-Rényi.
- **Graph matching: recovery.** Given (G_0, G_1) sampled from correlated Erdös-Rényi, find the π^* that minimizes the distance.

❑ Prior Work

➢ Only exponential time algorithms were known, *e.g., percolation*.

❑ Our Results

➢ **Graph similarity:** We give the *first polynomial time* algorithm.

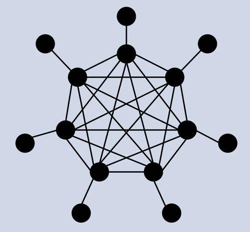
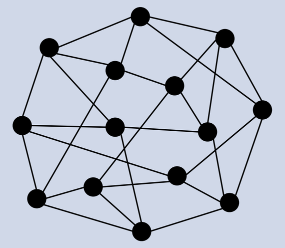
➢ **Graph matching:** We give the *first quasi-polynomial time* algorithm.

Paper	Algorithm	Runtime
Cullina & Kivayash	Info-theoretic	$\exp(O(n))$
Yartseva & Grossglauser	percolation	$\exp((1-\delta)n)$
This work	Subgraph matching	$n^{O(\log n)}$ *
Mossel & Xu	Seeded local statistics	$n^{O(\log n)}$ *

* The runtime does not work for all regimes. Ask me for more details!

❑ Our "Black Swan" Approach

➢ **Intuition:** Use a family of small graphs (a *flock of black swans*) as the features to compare (G_0, G_1).

A Swan	A Black Swan
	
The variance of #appearance is **large**.	✓ #appearance **concentrates** near exp.
Too many automorphisms.	✓ **Unique** automorphism.
Large overlap with other swans.	✓ **Small** overlap with other black swans.

➢ **Difficulties:** Construct a large family of black swans with the desiring properties.

❑ Algorithms

➢ **Graph similarity:** Use the *correlation of the black swan counts* to perform hypothesis testing.

- Let $\mathcal{H}$ be a family of black swans and $X_H(G)$ be the # of H's in G.
- Define the correlation polynomial:

$$P_{\mathcal{H}}(G_0, G_1) = \frac{1}{|\mathcal{H}|} \sum_{H \in \mathcal{H}} (X_H(G_0) - \mathbb{E}_G X_H(G))(X_H(G_1) - \mathbb{E}_G X_H(G)) .$$

- **(Correlated Erdös-Rényi):** $|P_{\mathcal{H}}(G_0, G_1)|$ is large.
- **(Independent Correlated Erdös-Rényi):** $|P_{\mathcal{H}}(G_0, G_1)|$ is small.

➢ **Graph matching:** For each vertex v, the black swan family gives a *signature vector* according to the position of v in each swan.

- **(Partial assignment):** The uniqueness of each swan guarantees the signature vector from G_0 and G_1 of the same vertex being close. This holds w.h.p. for many vertices and give a partial assignment.
- **(Boosting):** Use the partial assignment as the seeds and generate a full permutation that matches G_0 and G_1.

❑ Future Directions

➢ **For theorists:** (i) Improve the runtime, (ii) construct black swans for a larger range of parameters, and (iii) computational limitation.

➢ **For experimentalists:** Can our black swan approach guide practical algorithms for graph matching?

- Conference version:
- arXiv version: